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PERFORMANCE ASSESSMENT OF OPTICAL PARTICLE COUNTERS

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ABSTRACT

An Optical Particle Counter (OPC) is a commonly used instrument for calculating combustion efficiency as well as measuring air quality. An evaluation of the literature on OPC's shows many measurement inconsistencies. Several papers evaluate OPC's by comparing them with research grade instruments and generally recommending correction coefficients. This leads to confusion on the actual accuracy of OPC's. Little to no articles evaluate OPC accuracy by using their fundamental operating principles. This objective of this article is to address the question of OPC accuracy by using Mie Scattering Theory to create a theoretical photodetector response, which is then used to calculate the diameter of the detected particle. This information is used to evaluate the calculated diameter error inherent in OPC designs. It was found that in certain configurations a diameter error as little as 50% may exist; whereas in other configurations this error may be as high as 425%.

Keywords: Optical Particle Counter, Mie Scattering Theory, PM, Particulate Matter

NOMENCLATURE

q_d	Differential detector power (W)
$G_{i,d}$	Irradiation on differential detector $\left(\frac{\mathrm{W}}{\mathrm{m}^2}\right)$
A_d	Differential area of detector (m^2)
$I_{i,d}$	Intensity on differential detector $\left(\frac{\mathrm{W}}{\mathrm{m}^2\, sr\, \mu m}\right)$
$\theta_{i,d}$	Angle of irradiation on differential detector
$d\Omega$	Differential solid angle
$\Delta\Omega_{d-p}$	Finite solid angle as viewed from detector (sr.)
I_s	Intensity scattered from particle $\left(\frac{\mathrm{W}}{\mathrm{m}^2\, sr\, \mu m}\right)$
$\vec{R}_{12}$	Vector distance from particle to detector (m)
A_p	Projected area of particle (m^2)
P	Power of incident wave (W)
d	Distance from light source to particle (m)
x	Size parameter $\left(\frac{2\,\pi\, r_p}{\lambda}\right)$
r_p	Radius of particle (m)
λ	Wavelength of incident light (m)
k	Wavenumber $\left(\frac{2\,\pi}{\lambda}\right)$
θ	Scattering angle
S_1	Parameter in amplitude scattering matrix
S_2	Parameter in amplitude scattering matrix
q_{Total}	Total detector power

1. INTRODUCTION

OPC's are frequently used in the assessment of biomass cookstoves. Across the world, approximately 3 billion people burn biomass fuels in inefficient and highly polluting cookstoves each day (World Health Organization, 2019). Incomplete combustion of biomass produces toxic fumes and particulate matter which increases the risk of cancer, damages immune systems and irritates airways. The poor indoor air quality caused by biomass combustion is estimated to cause approximately 4 million premature deaths annually (World Health Organization, 2019).

Significant amounts of soot (black carbon) are produced in the fuel-rich conditions that typically occur in open fires and cookstoves. In addition to degrading indoor air quality, soot produced by biomass combustion is a significant contributor to global climate change. Improved cookstoves – economical, fuel-efficient, and clean-burning – have the potential to reduce anthropogenic climate change and to minimize the negative health effects associated with biomass combustion. It is clear that the design, optimization, distribution, adoption and sustained use of improved cookstoves is an imperative for both developed and developing nations.

In order to attain such goals, it is necessary to have the capability to conduct accurate analyses of the combustion performance of such stoves. These analyses may be used to further improve biomass cookstoves as previously stated and make product recommendations to the consumers of these cookstoves. Organizations that currently conduct such analyses often employ the use of low-cost instruments such as OPC's (under $100). In some examples, the manufacturers of these low-cost OPC's may provide limited information on the precision of the sensor. However, most if not all, come with no information on their true accuracy and measurement uncertainty. In nearly all circumstances, if accuracy information is provided it is simply a comparison of values to a "trusted" sensor. In such circumstances, no evidence is provided in order to show that the trusted sensor is correct other than by simply stating that it has been calibrated.

On further inspection of the academic literature on OPC's it is found that a similar situation exists. There are numerous articles comparing the results of OPC's to research grade instruments. A common example is that of the Tapered Element Oscillating Microbalance (TEOM). Many articles exist comparing several makes and models of OPC's to TEOM's. A correction coefficient is often found to make the OPC data fit as closely as possible with that of the TEOM. However, in little to no articles is evidence ever provided to prove that the TEOM is calculating particle size correctly. In such papers, the TEOM is merely trusted as accurate only because it is labeled as a research grade instrument. An evaluation of these types of articles makes it clear that the accuracy of OPC's must be evaluated on a more fundamental level.

This aforementioned evaluation is necessary in order to better understand the true accuracy of OPC's without inducing the error inherent in simply comparing sensors against sensors of unknown uncertainty. An assessment of the accuracy of low-cost OPC's will greatly benefit the organizations that utilize them in analyzing biomass cookstoves. Any person or organization employing the use of OPC's will be able to better characterize particulate matter concentration and combustion performance of biomass cookstoves. This will allow engineers to further develop improved cookstoves that specifically reduce current household emission concentrations which will improve global human health as well as reduce anthropogenic climate change.

2. METHODS

The goal of this article is to assess the validity of using cheap OPC hardware to calculate particle size diameter and the measurement uncertainty associated with these basic principles. An OPC is that a single particle is illuminated by a monochromatic beam of light. The particle scatters light onto a photodetector where an Integrated Circuit (IC) is used to measure the current or voltage produced by the photodetector. This value is then used to calculate a particle size. This article will explore the case where the measured value is a current. This will be accomplished by first considering the governing equations to calculating the power incident on the photodetector. Each term will be derived and/or substituted until an equation representative of the chosen system is obtained.

The equations derived in this article are in reference to the coordinate system shown in Fig. 1 which is also the same coordinate system projected onto Fig. 2. In order to evaluate the

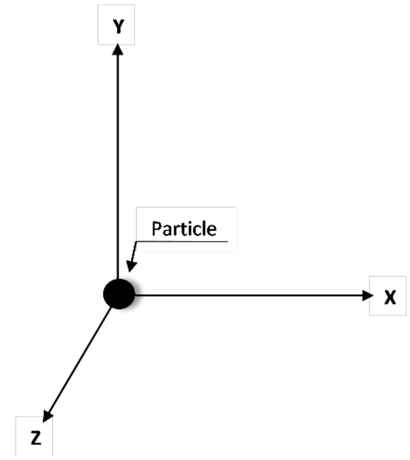

Fig. 1. Shows the global coordinate system used in this article, where a single particulate is at the origin

effectiveness of low-cost OPC's, the general geometry of the considered system is specified in Fig. 2. The following equations are first derived for a differential square element on the photodetector, after which a MATLAB script is used to sum all the differential elements for a given size parameter. This allows for a prediction of the power incident on the photodetector which may then be related to the current produced using a linear relationship. The total power incident on the photodetector is the incident irradiation multiplied by the area of the detector.

$$q_d = G_{i,d} A_d \tag{1}$$

In the general designs of these cheap OPC's the photodetector is often placed relatively close to the particle such that the field of view of the detector, as viewed from the particle, is sufficiently large. This article will consider light scattered from the particle, onto a differential element of the photodetector in order to use the solid angle approximation. The differential powers will then be summed in order to obtain a total detector power.

The irradiation incident on a differential element of the photodetector is defined using the definition of irradiation such that:

$$G_{i,d} = \int_{2\pi} I_{i,d} \cos(\theta_{i,d})\, d\Omega \tag{2}$$

Where P is the power of a monochromatic light incident on the particle of interest, x is the size parameter, and d is the distance from the light source to the particle. The parameters $S_1(\theta)$ and $S_2(\theta)$ are parameters in the amplitude scattering matrix, which are calculated in a script utilizing Mie Scattering Theory

(Markowicz). In consideration of a differential element of the photodetector, the intensity ($I_{i,d}$) is approximated as constant over that solid angle and the angle ($\theta_{i,d}$) of irradiation incident on the differential photodetector is also a constant. As such Eq. 2 becomes:

$$G_{i,d} = I_{i,d}\cos(\theta_{i,d}) \int_{2\pi} d\Omega \tag{3}$$

Considering the differential element of the photodetector, it may be said of the incident intensity:

$$I_{i,d} = \begin{Bmatrix} I_s & \Delta\Omega_{p-d} = d\Omega \\ 0 & \Delta\Omega_{p-d} \neq d\Omega \end{Bmatrix} \tag{4}$$

Equations 3 and 4 lead to:

$$G_{i,d} = I_s\cos(\theta_{i,d}) \int_{\Delta\Omega_{d-p}} d\Omega \tag{5}$$

Which may then be solved, such that:

$$G_{i,d} = I_{i,d}\cos(\theta_{i,d})\,\Delta\Omega_{d-p} \tag{6}$$

Where the solid angle is defined as:

$$\Delta\Omega_{d-p} = \frac{A_p}{|\vec{R}_{12}|^2} \tag{7}$$

Note that there is no cosine in the formula for the solid angle since an approximation of a spherical particle has been made. $Cos(\theta_{i,d})$ may be found using the dot product, such that:

$$\cos(\theta_{i,d}) = \frac{-\vec{R}_{12}\cdot\hat{j}}{|\vec{R}_{12}|} \tag{8}$$

R_{12} in the preceding paragraph is found using coordinates located at the center of each differential element of the photodetector. Combining Eq.'s 2 - 8 yields the following results:

$$G_{i,d} = \frac{I_{i,d}(-\vec{R}_{12}\cdot\hat{j})A_p}{|\vec{R}_{12}|^3} \tag{9}$$

It is necessary to define the intensity incident on the differential photodetector ($I_{i,d}$) which is equal to the intensity scattered from the particle in the direction of the differential photodetector as indicated in Eq. 4. This value is calculated in the following equation (see Appendix A for derivation):

$$I_s = \frac{P}{8\pi^2 d^2 x^2}(|S_1(\theta)|^2 + |S_2(\theta)|^2) \tag{10}$$

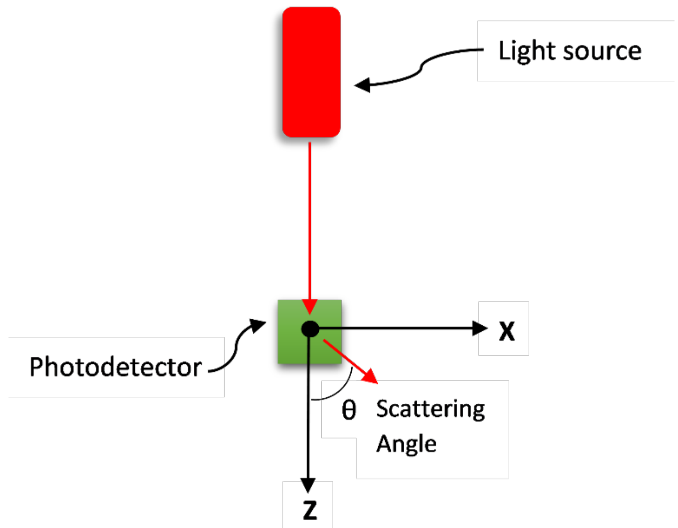

Fig. 2. Shows a 2-Dimensional top-down view of the system under consider. As can be seen, the scattering angle θ is formed in the xz plane. The scattering angle is 0° when it is in the forward direction +z.

Substituting Eq. 10 into Eq. 9 yields:

$$G_{i,d} = \frac{P\,(-R_{12}\cdot\hat{j})A_p}{8\pi^2 d^2 x^2 |\vec{R}_{12}|^3}(|S_1(\theta)|^2 + |S_2(\theta)|^2) \tag{11}$$

Substituting Eq. 11 into Eq. 1 results in the following equation:

$$q_d = \frac{P\,(-R_{12}\cdot\hat{j})A_p A_d}{8\pi^2 d^2 x^2 |\vec{R}_{12}|^3}(|S_1(\theta)|^2 + |S_2(\theta)|^2) \tag{12}$$

This equation may be further simplified by substituting in the projected area of the sphere (circle).

$$q_d = \frac{P\,(-R_{12}\cdot\hat{j})r_p^2 A_d}{8\pi d^2 x^2 |\vec{R}_{12}|^3}(|S_1(\theta)|^2 + |S_2(\theta)|^2) \tag{13}$$

The power q_d is calculated on each differential element of the photodetector after which a simple summation of all the differential powers detected yield the total power predicted for a given size parameter such that:

$$q_{Total} = \sum_{i=1}^{n} q_d \tag{14}$$

$$q_{Total} = \sum_{i=1}^{n} \frac{P\,(-R_{12}\cdot\hat{j})r_p^2 A_d}{8\pi d^2 x^2 |\vec{R}_{12}|^3}(|S_1(\theta)|^2 + |S_2(\theta)|^2) \tag{15}$$

It is important to note that the power, P, is the power of the incident wave. For example, a Light Emitting Diode (LED) may be rated at 20 mW of power consumption but an efficiency must be taken into account by the reader to determine the power P incident on the particle.

A MATLAB script was used to evaluate Eq. 13 and subsequently Eq. 15. These equations were evaluated for all size parameters of interest as well as all angles $0 - \pi$ according to the coordinate system defined in Fig. 2. Graphs were then created in order to give a graphical representation of the results. A general idea of the code logic is given in Fig. 3.

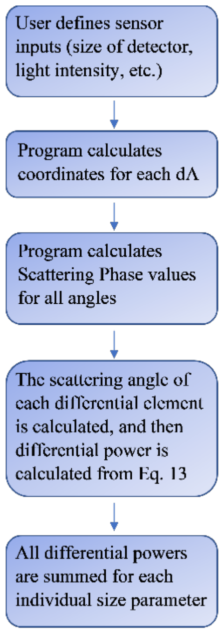

Fig. 3. Shows the essence of the programming logic used in the MATLAB script in order to calculate a theoretical photodetector response. The same logic is used for all detector arrangements considered in this paper.

3. RESULTS AND DISCUSSION

Physical dimension measurements were taken of a generic, low-cost OPC in order to further investigate the effectiveness of the technology currently being utilized in sizing small particles. This article particularly seeks to address the question of how accurate OPC's are in the $PM_{2.5}$ particle size bin. This is the size bin most harmful to human health, as well as a common bin used in combustion diagnostics (Environmental Protection Agency). Input parameters based on the aforementioned measurements were:

Power (P)	0.015 W	Differential Area, (A_d)	$2.3*10^{-11}$ m
Distance, source to particle (d)	0.013 m	Distance (in y), particle to detector	0.0022 m

Table 1. Shows the user-defined variables that are used to obtain the following information.

These input parameters, along with the methodology outlined in the methods section, produce the graph in Fig. 4. The response of the detector was modeled as a silicon photodetector with a response of 0.7 A/W.

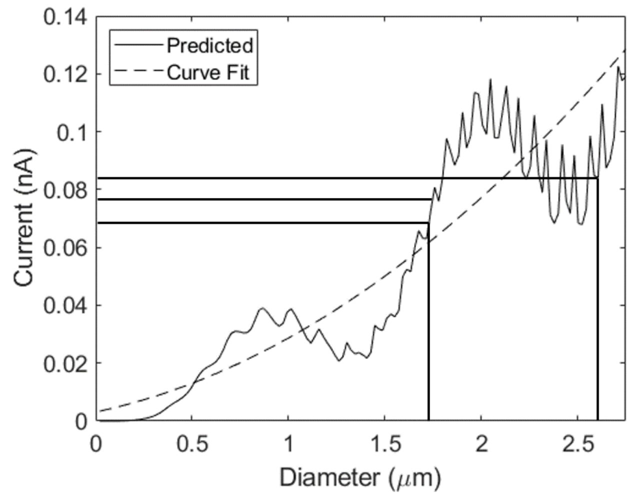

Fig. 4. Shows possible measurement uncertainty for $PM_{2.5}$ or less. In this case, the predicted error bars in the measuring the current ($\pm 5\%$) show that the measurement is in the nano-Amps range.

As can be seen in this figure, the calculated particle diameter may be off by as much 50% (2.63 μm and 1.75 μm). It is also apparent from the graph that there is little sensitivity in this range thus proving even harder for these OPC's to accurately size small particles. A heatmap was generated for the total photodetector area to evaluate which areas of irradiation had the largest effect on the photodetector response.

It was found that angles closest to the forward scattering ($\theta = 0^{\circ}$) direction had the largest affect on the total photodetector response. This is seen in Fig. **5** where the detector is laid such that it aligns with the coordinate system defined in Fig. **2**. Although Fig. **5** is for a single size parameter, the trend remains the same for all particle diameters below 2.5 μm.

A secondary OPC design was investigated as well. This design differed to the previous one in that the photodetector was placed at a much larger distance from the particle and in a different plane. A schematic is presented in Fig. **6**.

It was found that this design not only detected a current that was an order of magnitude less than the first design, but also when used to calculate $PM_{2.5}$, it had an enormous measurement uncertainty. It was found that the measurement uncertainty in this configuration could reach as high as 425% (2.63 μm and 0.5 μm) thus making this design much worse in measuring $PM_{2.5}$ than the previous design. A heatmap of the photodetector in this configuration was likewise produced and is shown in Fig. **8**. This design should not be used to measure $PM_{2.5}$ as there is virtually no accuracy or confidence in the actual size of the particle and as such a concentration could not be calculated with any confidence.

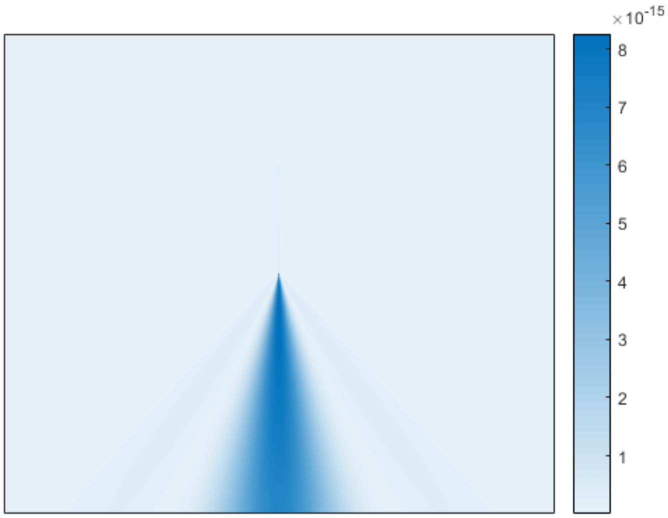

Fig. 5. Shows the heatmap of the photodetector for a particle diameter of 2.5 μm. The units of the scale are in nA.

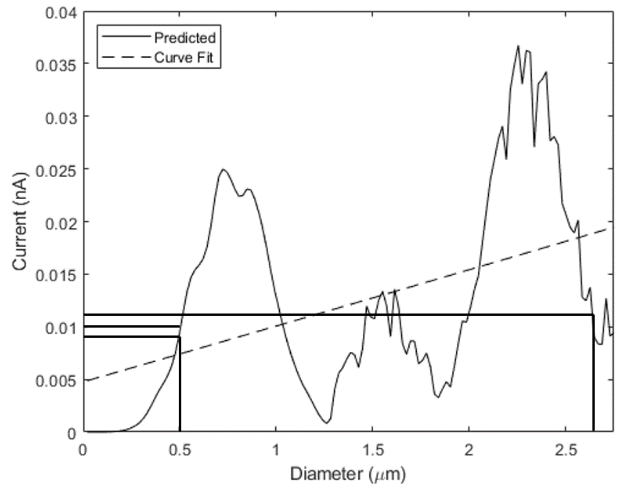

Fig. 7. Theoretical response of a silicon photodetector in the arrangement described in Fig. **6**. The measurement uncertainties in this configuration are extremely high.

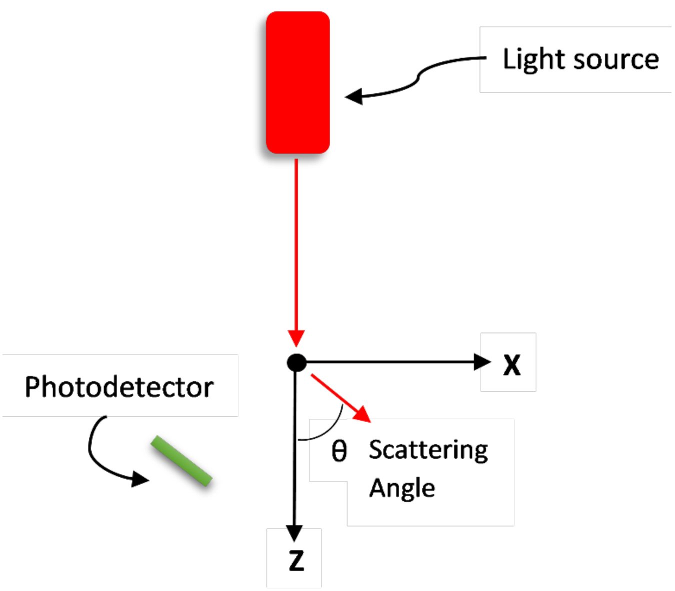

Fig. 6. Shows a schematic of the second OPC design considered in this article. The sensing element of the photodetector is into and out of the page.

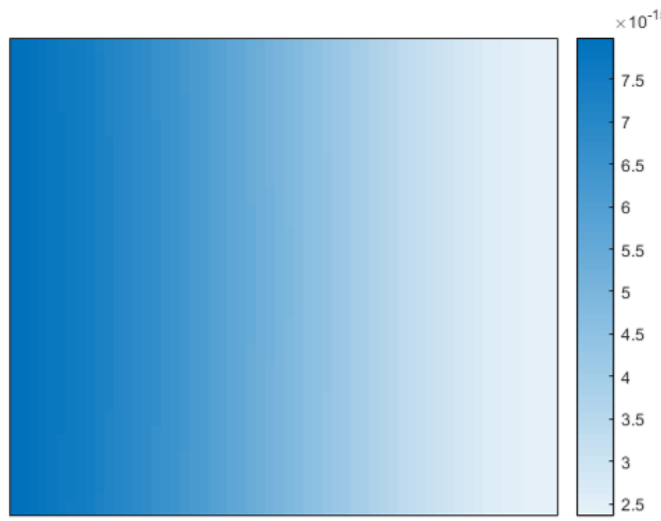

Fig. 8. A heatmap for the second OPC configuration is shown. This figure is looking at the photodetector as if from the particle, such that the left side is closer to the forward scattering angles than is the right side. The units are in nA.

In regard to future work, now that mathematical models have been produced and analyzed for two current OPC designs it would be important to modify the existing code to optimize placement of the photodetector or an arrangement of photodetectors in order to optimize the design and create a response curve that is at the least more nearly 1:1, as this would decrease the measurement uncertainty.

4. CONCLUSION

OPC's induce a significant amount of measurement uncertainty that come simply because of the basic principles they work off of. It is clear that some designs introduce more error than others. As such, it is important to validate an OPC design before using it for research, as such one may understand the results better knowing taking the uncertainty into account.

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