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ICE-CREAM CHURNING

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ABSTRACT

This paper deals with fairly simple, cylindrical coordinate fluid flow. It is the analysis of two different fluids undergoing cylindrical Couette flow. The method used was taking the full Navier-Stokes equations and deriving the ODE for the simplified case. The paper shows how ice-cream churning can linearly increase in difficulty as the cream solidifies.

NOMENCLATURE

Put nomenclature here.

INTRODUCTION

In my robotics class my teammate and I set out to churn ice-cream with a robot arm. The robot we used was fairly weak despite its size. We wanted to better understand the torque required to crank the ice-cream so we could ensure feasibility. This is our simplified problem statement:

How much torque is required to spin an ice-cream bucket and churn the interior ice-cream. The equivalent diameter of the water bucket, ice-cream bucket, and center shaft are respectively, .381m, .2286m, and .0254m. Assume infinite in Z and ignore end effects. There is water between the water bucket and the ice-cream bucket with a viscosity of 1750 * 1^-6 N*s/m^2. The ice-cream between the ice-cream bucket and the center shaft is .1 N*s/m^2. Assume laminar flow for both fluids.

Put body of the paper here:

The aim of this paper is to determine the way fluids act when they are being spun in opposite directions inside an ice-cream bucket. The boundary conditions are fairly simple as they are no slip wherever the ice-cream is in contact with the solid surfaces. The difficult part was found in the geometry itself was the difficult part. The interior is made up of paddles intended to create large amounts of mixing. This means that you have r, ϕ, and z momentum. You also have significant derivative of all the velocities in all the directions. Therefore I would have to solve the full Navier-Stokes equations. This is something only solvable by computer software. I altered the geometry for my problem and simplified it to have only a cylinder in the center instead of all the paddles. I made the cylinder larger so that it would hopefully accommodate the change in geometry for my analysis to be semi-accurate for finding the torque required to crank.

Assumptions that I made were that gravity was not in the θ direction, it was steady state flow, it isn't pressure driven flow, it is axisymmetric, and fully developed. With these equations the problem reduced to a second order, linear, second order differential equation.

$$\frac{d^2u_\phi}{dr^2} + \frac{d}{dr}\left(\frac{u_\phi}{r}\right) = 0$$

The analytical solutions was found through the method of separation of variables.

$$u_\phi = C_1 r + \frac{C_2}{r}$$

At this point it remains to find the coeficients C1 and C2. For my case I have a cylinder within a cylinder withing a cylinder. I also have two different fluids, ice-cream and ice water. I had to apply the three boundary conditions of no slip or $u_\phi = \omega r$ at all boundaries. Applying these Dirichlet conditions everywhere for the two fluids yielded the two equations for the different fluids.

$$u_\phi = -6.4403 * r + .0021/r$$

$$u_\phi = 3.5343 * r - 0.1283/r$$

These velocity profiles can be seen attached as *Figure* 1. As can be seen here the viscosity of the two fuids doesn't play a role in their velocities or velocity profiles because this is essentially Couette flow. The shear however does rely on viscosity of the fluid.

$$\tau = -\mu \frac{du_\phi}{dr}$$

Applying the two different viscosities I obtained the graph shown in *Figure 2*

$$\tau_{ice-cream} = -\mu_{ice-cream} \frac{du_\phi}{dr}$$

$$\tau_{ice-water} = -\mu_{ice-water} \frac{du_\phi}{dr}$$

I then multiplied the shears on all the surfaces to obtain a torque required to crank the ice-cream. As you crank ice-cream it crystalizes and solidifies and the fluid properties change. To accommodate for this I used a span of 0.1 to 1.0 Pa*s for the viscosity to model the change. The viscosity of the ice-cream is much more than that of ice water and therefore very quickly it makes much more difference to the torque required than the water.

CONCLUSIONS

It was determined from the calculations that when you are first starting to crank the ice-cream and is still basically cream, the torque required is 0.6099 N*m. By the time the viscosity has increased by 1 order of magnitude, the torque required is up to 6.2504 N*m. That is equivalent to 57 lb*in. The crank of the ice-cream bucket has a 7 in radius and therefore the force required to crank it is a little over 8 lbs.

For the robot that we used, this final torque is probably too high as it can only put out about 2 lb of force and the 8 lb isn't even the torque needed from the greatest viscosity of the ice-cream. The result is that while it is fun to churn a brobot into a probot and to crank ice-cream, it is probably too hard for the robot we used. My model needs further validation in comparison with other ice-cream cranking models but it seams fairly reasonable as I have often cranked ice-cream and the torque required changed drastically as can be seen here.

REFERENCES

[1] Flettner and Childs, 2011, "Rotating Cylinders, Annuli, and Spheres," http://www.homepages.ed.ac.uk/shs/Climatechange/Flettner%20ship/Childs%20Annuli.pdf

APPENDIX

Matlab Code for computing C1, C1, C3, C4 and plotting the graphs.

```
clear
close all
clc

% Subscripts cs, ic, wb reference center
shaft, ice-cream bucket, and water
% bucket respectivelyc
mu_icecream = [.1:.1:1]; % N*s/m^2
mu_water = 1750e-6; % N*s/m^2
h = .5; % height of icecream bucket
Dcs = .0254; % meters
Dic = .2286; % meters
Dwb = .381; % meters
r = [Dcs Dic Dwb]/2;
wcs = 2*pi; % m/s
wic = -2*pi; % m/s
wwb = 0; % m/s
w = [wcs wic wwb];

system_1 = [r(1) 1/r(1) w(1)*r(1);
            r(2) 1/r(2) w(2)*r(2)];
system_2 = [r(2) 1/r(2) w(2)*r(2);
            r(3) 1/r(3) w(3)*r(3)];
c1_c2 = rref(system_1);
c1 = c1_c2(1,3);
c2 = c1_c2(2,3);
c3_c4 = rref(system_2);
c3 = c3_c4(1,3);
c4 = c3_c4(2,3);

r1 = [r(1):.0001:r(2)];
r2 = [r(2):.0001:r(3)];
v_icecream = c1.*r1+c2./r1;
v_icewater = c3.*r2+c4./r2;
d_dr_icecream = c1-c2./r1.^2;
d_dr_icewater = c3-c4./r2.^2;

figure()
plot(r1,v_icecream,'--',r2,v_icewater)
title('Velocity profile inside and outside
ice-cream bucket')
legend('Ice-cream', 'Ice Water')
ylabel('v_{\theta} (m/s)')
xlabel('r (m)')
Tau_icecream =
zeros(length(d_dr_icecream),length(mu_icec
ream));
for i = 1:length(mu_icecream)
    Tau_icecream(:,i) = -
mu_icecream(i).*d_dr_icecream;
end
```

```
Tau_water = -mu_water*d_dr_icewater;
figure()
hold on
for i = 1:length(mu_icecream)
    plot(r1,Tau_icecream(:,i),'--')
end
plot(r2,Tau_water)
title('Shear of both fluids with
increasing \mu_{ice-cream} as a function
of r')
legend('.1', '.2', '.3', '.4', '.5', '.6',
'.7', '.8', '.9', '1.0', 'Ice Water')
ylabel('\tau (Pa)')
xlabel('r (m)')
Tau_total =
Tau_icecream(1)+Tau_icecream(end)+Tau_wate
r(1)
Torque_to_crank =
pi*(Tau_icecream(1,:).*Dcs+Tau_icecream(en
d,:).*Dic+Tau_water(1)*Dic)
figure()
plot(mu_icecream,Torque_to_crank)
title('required torque with increasing
\mu_{ice-cream}')
ylabel('Torque (N*m)')
xlabel('\mu_{ice-cream} (Pa*s)')
```

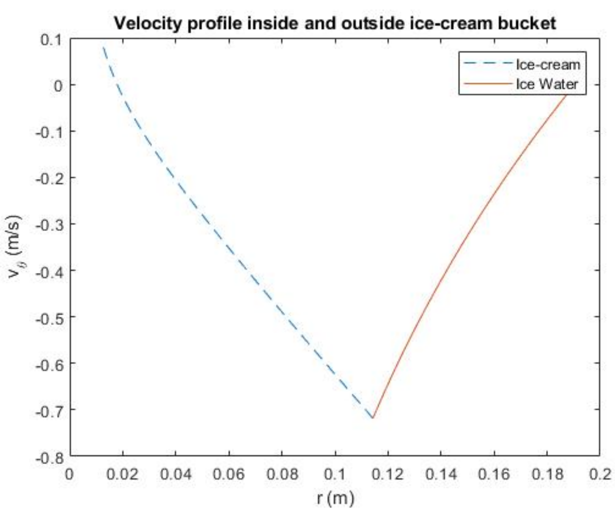

Figure 1. Velocity profile of ice-cream inside the bucket and water outside the bucket. The center shaft spins the opposite direction from the ice-cream bucket.

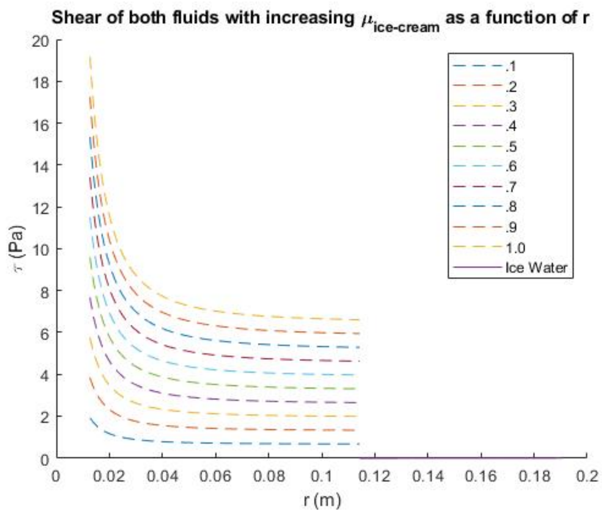

Figure 2. Shear profiles for the different viscosities of ice-cream and on the very bottom right you can see the shear from the water. Water shear is nearly negligible the longer you crank. To get the shear force for any time you take the leftmost shear of the ice-cream graphs times center shaft surface area plus rightmost shear of ice-cream graph times ice-cream bucket surface area plus leftmost shear of ice water graph times the surface area of the ice-cream bucket.

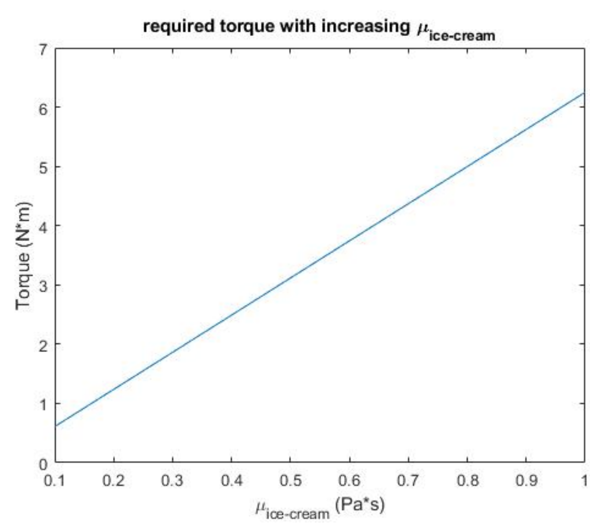

Figure 3. Required Torque plotted against increasing viscosity of ice-cream due to freezing.

