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TRANSIENT THERMAL MODEL OF A BAKING CAKE

Tracy Vogel

Mechanical Engineering Department
Brigham Young University
Provo, Utah 84602
tracy.vogel99@gmail.com

ABSTRACT

Cake presents an interesting heat tranfer problem as it has a constantly changing shape and material profile as it cooks. Matlab has many functions to play with in creating this model but does not allow for a growing model. In this report, I will be evaluating multiple aternative modelling methods in Matlab to identify the most likely areas to burn when baking a cake.

INTRODUCTION

A round cake starts out as a thin disk of batter which thickens and becomes less dense as it is heated. Porous cake doesn't transfer the heat as well as the batter originally did, so as the density reduces, so does the heat transfer coefficient. I will be creating a simplified boundary-value model of this problem to identify the most likely area to burn and how it compares to the rest of the cake to reduce the proportion of cakes burnt in the future.

METHODS

Gathering Initial Data:

There are far too many factors in a cake to account for everything in a single model. Recipe, imperfect mixing, hotspots, even elevation is known to affect baked goods significantly. I will instead be constructing my model from one cake, which I baked myself and took my data from.

The initial depth of the batter was about 1.5 cm and room temperature (20 C°). The final cake was 4 cm tall. The cake rose in a rough approximation of a logistic function. This data was gathered using a timelapse video from a camera mounted outside the oven, as shown in figure 1. The finished cake was around 95 C° average, which acts as a sanity check to assure my model is working well and was used through multiple rounds of testing. The cake baked for an additional 20 minutes (40 total) but the data was uninteresting past the 20-minute mark and so has been omitted.

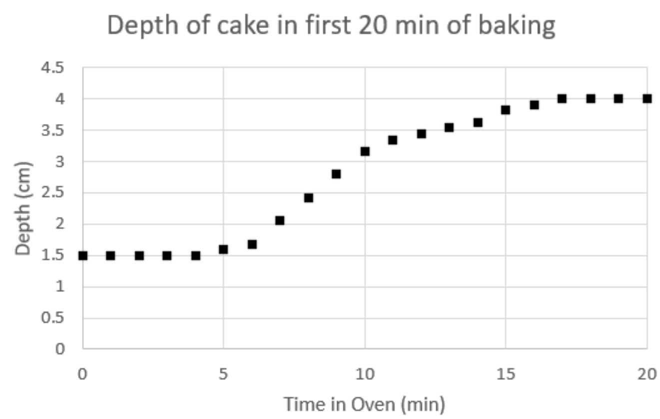

Figure 1: Rising cake experimental dataset

Matlab Program Functionality:

Matlab is one of the top programs in engineering. Its axisymmetric thermal modelling capabilities proved invaluable to this project, but even this behemoth has some drawbacks. Starting from an old temerature profile requires the mesh to remain the same across the two, which in Matlab means it must be the same shape. There are two ways to approach this: Change the function for the heat transfer and image, or hard code your own way.

Initial model:

My first attempt was a model which re-generated the mesh for each point in time with a different height of cake. However, this new mesh did not allow the old data to map to it. I tried hard coding a workaround by pulling the mesh size for each time step, then adding logical values to the center of the prevoious mesh to grow it to the new size, but the program would not accept the result as a valid initial condition. Starting from an old temperature profile simply isn't possible if the geometry changes.

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Final model:

Using a single shape to model dynamic geometry is doable so long as the material properties alter to reflect the changing state of the model. Luckily, the amount of material in the cake doesn't change significantly. It inflates, which changes the density of the material, but that material is still there, so for the purposes of the model, I treated the density as a constant. This way it takes the same amount of energy to change temperature. I did, however, alter the conductivity in hotter areas. As the cake inflates, it won't conduct heat as well due to changes in both porosity and distance traveled, and it will inflate more in hotter areas. To model this, I set conductivity decrease linearly in hotter areas using the coefficients from O. D Baik et al as endpoints.

Most cakes are also moist. This means they have an in-buit cooling system at exactly 100 degrees celcius. In essence, the batter heats a lot slower around this temperature. Given enough computing power, this could be modelled by an increase in specific heat using a heaviside function multiplied by water's heat of vaporization (2,260,000 J/kg), to the Specific Heat, but this proved too much for my computer so I opted to simplify the model to an oil-heavy cake, which will have no such cooling source. This should have the bonus effect of making it much clearer what areas of the cake get hottest fastest, as there is no drive toward a low equilibrium.

The boundary conditions are all convective in an oven, however, the pan poses a unique issue. It acts first as an insulator, then as a conductor as it heats up. In this model I idealized this as a linear function. Convection coefficients, specific heat, and densities were all found by external research. (O. D Baik, S Grabowski, Majzoobi)

The final animation is achieved by pulling one frame from each minute of the baking process. To show the inflation visually without altering the geometry itself (for reasons mentioned earlier), I manually set the y limits to shrink for each iteration. This makes a nice illusion of rising to make the model more comprehensible.

CONCLUSIONS

The part of the cake most likely to burn early is the top of the rim around the outer edge. This is the best indicator of the 'doneness' of the cake interior. However, if left for a long time, the bottom will likely burn worse due to the ever-increasing temperature of the pan.

REFERENCES

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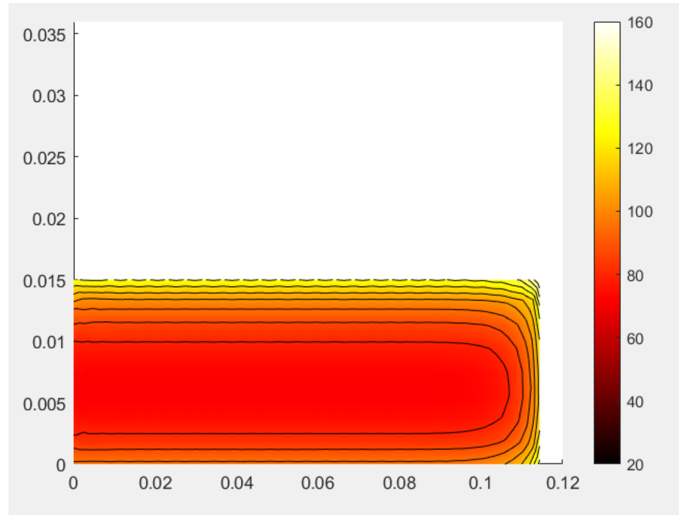

Figure 2: An intermediate frame of the cake's temperature profile as it cooks

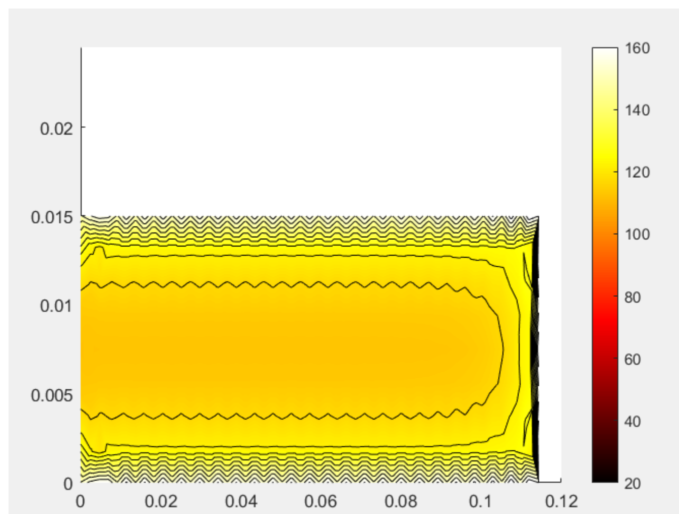

Figure 3: The cake's final temperature profile

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APPENDIX

Matlab Code for Final Model

```
%values found in research
densityavg = 1.15*(10.^6); %g/m^3
%https://www.researchgate.net/figure/Density-and-Bostwick-number-of-the-cake-batter-containing-different-levels-and-different_tbl1_281733110#:~:text=Increasing%20bran%20quantity%20and%20particle,to%205.00%20cm.%20...&text=...%20et%20al.
ThermCondavg = .25; %W/mk
SpecHeatavg = 2.517; %J/gK
%http://lib3.dss.go.th/fulltext/Journal/Journal%20of%20food%20science/1999%20v.64/no.2/jfsv64n2p295-299ms1308%5B1%5D.pdf


%Variables from my cooking setup
ti = 20; % initial temp C   %room temp
tf = 95; % final temp C
ovenTemp = 163; %C
thicki = 1.5/100; % initial cake depth in m
thickf = 4.0/100; % final cake depth in m
D = 22.86/100; %cake diameter in m
time = 40; %minutes baked
%thickc = thicki+(thickf-thicki)*t/time;

CTE = thickf/thicki/(tf-ti); % working coefficient of thermal expansion




cake = createpde("thermal", "transient-axisymmetric");

%geometry
gd = [3,4, [0, 0, D/2, D/2, 0, thicki, thicki, 0]]'; %[rectangle, 4 sides, x of corners, y of corners]
g = decsg(gd);
geometryFromEdges(cake,g);

%boundaries
topCC = 30; %W/m^2K
%http://lib3.dss.go.th/fulltext/Journal/Journal%20of%20food%20science/1999%20v.64/no.4/jfsv64n4p688-694ms4258[1].pdf
panCC = @(location, state)state.time/60;
thermalBC(cake, 'Edge', 2, "ConvectionCoefficient", topCC, "AmbientTemperature", ovenTemp);  %top
thermalBC(cake, 'Edge', 3, "ConvectionCoefficient", panCC, "AmbientTemperature", ovenTemp); %outer ring
thermalBC(cake, 'Edge', 4, "ConvectionCoefficient", panCC, "AmbientTemperature", ovenTemp);  %bottom

%initial condition
thermalIC(cake,ti);

%material properties
TCval = @(location,state)ThermCondavg - 0.002*(state.u-ti);
MDval = @(location,state)densityavg;
SHval = @(location,state)SpecHeatavg;% + 2260*heaviside(state.u-100); %2260 is water heat of vaporization <->20
thermalProperties(cake,"ThermalConductivity",TCval, "MassDensity",MDval,"SpecificHeat",SHval);

%output
mesh = generateMesh(cake);
tlist = [0:40]*60;
thermalResults = solve(cake, tlist);
T = thermalResults.Temperature;

for t = 1:time+1

pdeplot(cake,"XYData",T(:,t),"Contour","on","ColorMap","hot")
    clim([20,160]);
    ylim([0,.045-(t*.0005)]);
    pause(.1);
end
```


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