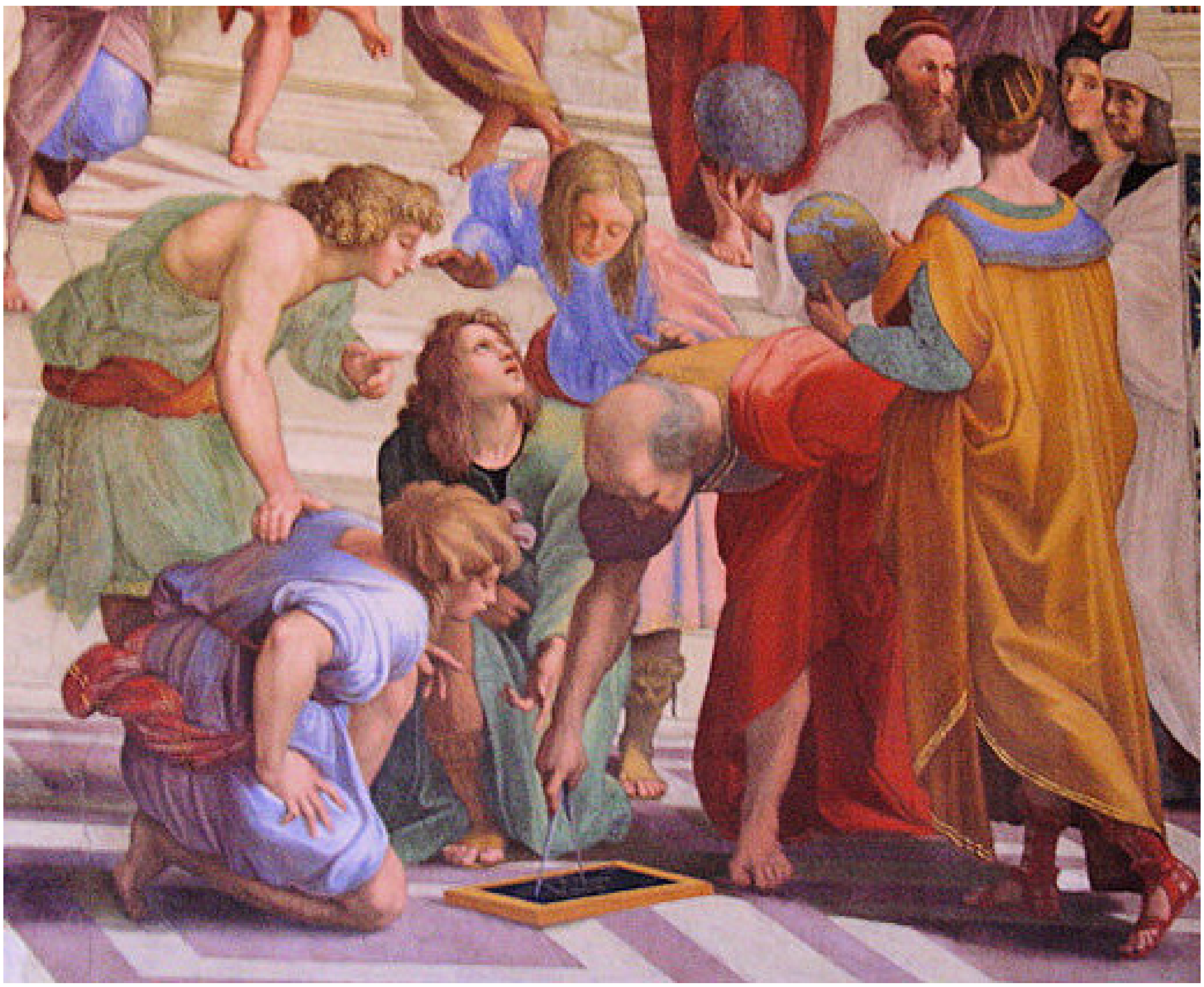
ME-505 2017 **FINAL EXAM**

1. No time limit. Exam is due before 3:00 pm Thursday, Dec 21 in MARB-214 (under the door).
2. *Please write neatly and show all your work to receive full credit*.
3. Computer and standard math software can be used only as the supportive tools.
4. Class notes, class web-site, and regular math books can be used.
5. No set of rules can cover all possible situations – be reasonable – do only what you believe is proper.



ME-505 Fall 2017	**Final Exam**:	Score: /40
Grading: Problem 1 (10 pts), Problem 2 (30 pts: 5 pts for each question)		

1. Let $D \subset \mathbb{R}^3$ be a ***domain*** in the Euclidean space. Show that if any open ball $B(\mathbf{r}, R)$ with the center at the point $\mathbf{r} \in \mathbb{R}^3$ and of any radius $R > 0$ includes points both from D and $\mathbb{R}^3 \setminus D$, then $\mathbf{r}$ is a boundary point of D.

2. Sturm-Liouville problem and integral transform method.

a) Consider the differential operator L applied to the function $y(r)$:

$$Ly(r) \equiv \frac{1}{r}\frac{d}{dr}\left(r\frac{dy}{dr}\right)-\frac{\nu^2}{r^2}y\,, \qquad 0<r_1<r<r_2\,, \quad \text{where } \nu \ge 0 \text{ is some fixed parameter.}$$

Find the solution of the eigenvalue problem for the given operator L:

$$Ly(r) = \lambda y$$

subject to the boundary conditions:

$$y(r_1)=0$$

$$y(r_2)=0\,.$$

b) Use the found eigenfunctions y_n for the Fourier series representation of the function $f(r),\ r_1 \le r \le r_2$.

Demonstrate it for the case $f(r)=H(r-r_1)-H\left(r-\frac{r_1+r_2}{2}\right)$ with $r_1=1,\ r_2=3$ and parameter $\nu=0$.

c) Construct the Finite Fourier Transform and the corresponding inverse transform based on the found eigenfunctions:

$$\overline{u}_n = \quad \Im\{u(r)\} \quad = \int_{r_1}^{r_2} u(r)y_n(r)\,p(r)\,dr \ =$$

$$u(r) = \ \Im^{-1}\{\,\overline{u}_n\} \quad =$$

d) Let the function $u(r),\ r_1 \le r \le r_2$ satisfies the boundary conditions

$$u(r_1)=f_1$$

$$u(r_2)=f_2\,.$$

Derive the result of application of the constructed integral transform (operational property) to:

$$\Im\left\{\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)-\frac{\nu^2}{r^2}u\right\} =$$

e) Apply the constructed integral transform for solution of the following initial-boundary value problem

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)+q(r,t)=\frac{1}{a^2}\frac{\partial u}{\partial t}\,, \qquad t>0\,, \quad r_1<r<r_2$$

$$u(r_1,t)=f_1(t)=f_1\cdot \sin\omega_1 t \qquad t>0$$

$$u(r_2,t)=f_2(t)=f_2\cdot \sin\omega_2 t \qquad t>0$$

$$u(r,0)=u_0(r) \qquad r_1 \le r \le r_2$$

f) Find the solution for the case: $r_1=1\,,\ r_2=3\,,\ a=0.2\,,\ f_1=1.0\,,\ f_1=3.0\,,\ \omega_1=0.5\,,\ \omega_2=1.0\,,\ q=0\,,\ u_0=0\,.$

Visualize (attach or email the code).