$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{x}\frac{\partial u}{\partial x} + \frac{1}{x^2}u + \frac{\partial^2 u}{\partial z^2} + S(x,z,t) = \frac{1}{w^2}\left(\frac{\partial^2 u}{\partial t^2} + 2\gamma\frac{\partial u}{\partial t}\right)$$

(1)

$$Lu(x) = a_0(x)\,u'' + a_1(x)\,u' + a_2 u$$

$$a_0(x) = 1$$
$$a_1(x) = -1/x$$
$$a_2(x) = 1/x^2$$

$$-\alpha_1 u'(x_1) + \beta_1 u(x_1) =$$

$$u(x_1) = 0 \quad 0<z<M,\ t>0$$
$$u(x_2) = 0 \quad 0<z<M,\ t>0$$

$$Lu(x) \equiv \frac{1}{p(x)}\left[(ru')' + qu\right]$$

$$Lu(x) \equiv x\left[\left(\frac{u'}{x}\right)' + \frac{u}{x^3}\right]$$

$$p(x) = \frac{1}{a_0(x)}\exp\left(\int \frac{a_1(x)}{a_0(x)}\,dx\right)$$
$$= \exp\left(\int \frac{-1}{x}\,dx\right)$$
$$= \exp(-\ln(x))$$
$$= 1/x$$
$$q(x) = a_2(x)\,p(x) = 1/x^3$$
$$r(x) = a_0(x)\,p(x) = 1/x$$

(2) $(ry')' + (q + \mu^2 p)y = 0 \qquad x_1 < x < x_2$

$$Lu(x) \equiv x\left[\left(\frac{u'}{x}\right)' + \frac{u}{x^3}\right] = \lambda u$$

$$x\left(\frac{u''}{x} - \frac{u'}{x^2}\right) + \frac{xu}{x^3} - \lambda u = 0$$

$$u'' - \frac{u'}{x} + u\left(\frac{1}{x^2} + \mu^2\right) = 0$$

② $(ry')' + (q + \mu^2 p)y = 0 \quad x_1 < x < x_2$

$$Lu(x) \equiv x\left[\left(\frac{u'}{x}\right)' + \frac{u}{x^3}\right] = \lambda u$$

$$x\left(\frac{u''}{x} - \frac{u'}{x^2}\right) + \frac{xu}{x^3} - \lambda u = 0$$

$$\mu_n = -\lambda_n^2$$

$$u'' - \frac{u'}{x} + u\left(\frac{1}{x^2} + \mu^2\right) = 0$$

$$\frac{1-2m}{x} - 2\alpha = \frac{-1}{x} \rightarrow \alpha = 0, \; m = 1$$

$m = 1$
$\alpha = 0$
$p = 1$
$a = \mu$
$v = 0$

$$p^2a^2x^{2p-2} + 0^2 + \frac{0}{x} + \frac{m^2 - p^2v^2}{x^2} = \frac{1}{x^2} + \mu^2$$

$$p^2a^2x^{2p-2} = \mu^2 \rightarrow 2p-2 = 0 \rightarrow p = 1$$

$$a^2 = \mu^2 \rightarrow a = \mu$$

$$m^2 - pv^2 = 1 \rightarrow 1 - 1v^2 = 1 \rightarrow v^2 = 0 \rightarrow v = 0$$

$$\boxed{u = x\left[c_1 J_0(\mu x) + c_2 Y_0(\mu x)\right]}$$

$$u(x_1) = x_1\left[c_1 J_0(\mu x_1) + c_2 Y_0(\mu x_1)\right] = 0$$

$$u(x_2) = x_2\left[c_1 J_0(\mu x_2) + c_2 Y_0(\mu x_2)\right] = 0$$

system of eq. has non-trivial solution only if determinate of coefficients matrix is equal to zero, i.e.

$$J_0(\mu x_1)Y_0(\mu x_2) - J_0(\mu x_2)Y_0(\mu x_1) = 0 \quad , \quad x_1 = 1, \; x_2 = 4$$

Solved for eigen values in Matlab (see plot / code attatched to the end of this pdf):

$$\boxed{\mu_1 = 1.0244 \qquad \mu_2 = 2.0809 \qquad \mu_3 = 3.1322 \rightarrow \text{Eigen values}}$$

Eigenfunctions are defined by

$$u_n(x) = x\left[c_{1,n} J_0(\mu_n x) + c_{2,n} Y_0(\mu_n x)\right] = 0$$

$$@\, x = x_1 \quad c_{1,n} J_0(\mu_n x_1) + c_{2,n} Y_0(\mu_n x_1) = 0$$

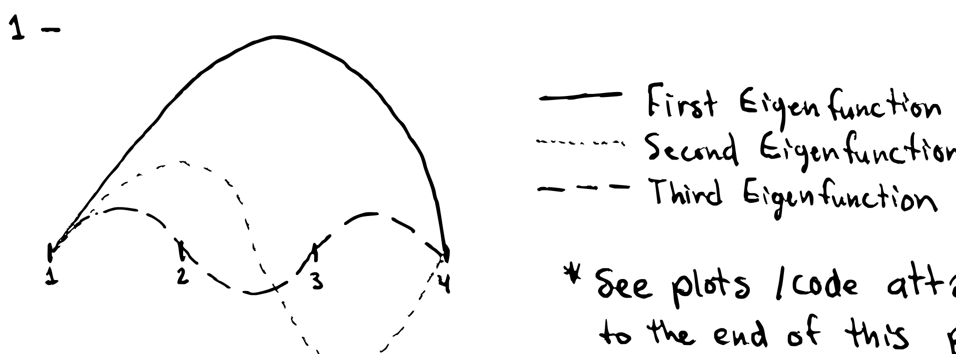
$$c_{1,n} = -c_{2,n} \frac{Y_0(\mu_n x_1)}{J_0(\mu_n x_1)}$$

thus

$$u_n(x) = x\left[-c_{2,n} \frac{Y_0(\mu_n x_1)}{J_0(\mu_n x_1)} J_0(\mu_n x) + c_{2,n} Y_0(\mu_n x)\right]$$

choose arbitrary co-efficients $c_{2,n} = J_0(\mu_n x_1)$

$$\boxed{\begin{array}{l}\text{Eigenfunctions:} \\ u_n(x) = x\left[J_0(\mu_n) Y_0(\mu_n x) - J_0(\mu_n x) Y_0(\mu_n)\right]\end{array}}$$



* See plots / code attatched to the end of this pdf

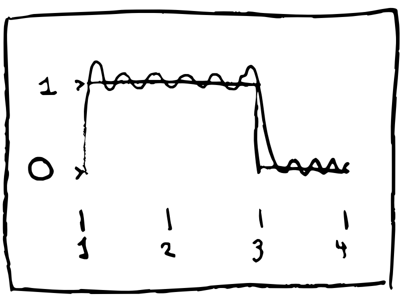
Weighted Inner Product: $(u, v)_p = \int_a^b u(x)\, v(x)\, p(x)\, dx$

$$\text{Norm} = \|u_n(x)\| = \left[\int_a^b u^2(x)\, p(x)\, dx\right]^{1/2} \qquad p = \frac{1}{x}$$

$$\text{Norm}^2 = \|u_n(x)\|^2_{p=1/x} = \int_{x_1}^{x_2} u_n^2(x) \frac{1}{x}\, dv$$

$$\boxed{\text{Norm}^2 = \int_1^4 \frac{u_n^2(x)}{x}\, dx}$$

③ $f(x) = 1 - H(x-3) = \sum_{n=1}^{20} a_n \frac{u_n(x)}{\|u_n(x)\|_p^2}$



$$a_n = \int_1^4 f(x)\, u_n(x) \frac{1}{x}\, dx$$

* See plots / code attatched to the end of this pdf

④ Direct Transform

$$\mathcal{J}\{u(x)\} = \bar{u}_n = (u, u_n)_p = \int_1^4 u(x)\, u_n(x)\, p(x)\, dx$$

$$u_n(x) = x\left[J_0(\mu_n) Y_0(\mu_n x) - J_0(\mu_n x) Y_0(\mu_n)\right]$$

$$p(x) = 1/x$$

Inverse Transform

$$\mathcal{J}^{-1}\{\bar{u}_n\} = u(x) = \sum_{n=1}^{\infty} \bar{u}_n \frac{u_n(x)}{\|u_n(x)\|_p^2}$$

$$\|u_n(x)\|_p^2 = \int_1^4 \frac{u_n^2(x)}{x}\, dx$$

⑤ D-D

$$J\{Lu\} = -\mu_n^2 \bar{u}_n - y_n'(x_2) r(x_2) f_2(\xi) + y_n'(x_1) r(x_1) f_1(\xi)$$

since $f_1 = f_2 = 0$

$$J\{Lu\} = -\mu_n^2 \bar{u}_n$$

⑥ Finite Integral Transform - X

$$\underbrace{\frac{\partial^2 u}{\partial x^2} - \frac{1}{x}\frac{\partial u}{\partial x} + \frac{1}{x^2} u}_{Lu_{x}\text{, by def'n in part 1}} + \frac{\partial^2 u}{\partial z^2} + S(x,z,t) = \frac{1}{w^2}\left(\frac{\partial^2 u}{\partial t^2} + 2\gamma \frac{\partial u}{\partial t}\right)$$

where:
$$\bar{S}_n(z,t) = \int_1^4 S(x,z,t)\, u_n(x)\, p(x)\, dx$$

transformed equation:

$$-\mu_n^2 \bar{u}_n + \frac{\partial^2 \bar{u}_n}{\partial z^2} + \bar{S}_n(z,t) = \frac{1}{w^2}\left(\frac{\partial^2 \bar{u}_n}{\partial t^2} + 2\gamma \frac{\partial \bar{u}_n}{\partial t}\right)$$

Finite Integral Transform - Z

$Lu_z \equiv \frac{\partial^2 u}{\partial z^2}$, already in self-adjoint form, $p(x) = 1$, $q(x) = 0$, $r(x) = 1$

SLP with D-D , $X_m = \sin(\lambda_m z)$, $\lambda_m = \frac{m\pi}{L}$

$\|X_m\|^2 = L/2$

Integral Transform: $\bar{u}_m(t) = \int_0^M u(z,t)\, X_m(z)\, dz$

Inverse Transform: $u(z,t) = \sum_{m=1}^{\infty} \bar{u}_{n,m} \frac{X_m(z)}{\|X_m\|^2}$

Operational property:

$$J_z\{Lu_z\} = -\mu_m^2 \bar{U}_m - y_m'(x_2) r(x_2) f_{2,m}(\xi) + y_m'(x_1) r(x_1) f_{1,m}(\xi)$$

since $f_{2,m} = f_{1,m} = 0$

$$J_z\{Lu_z\} = -\mu_m^2 \bar{U}_m$$

Transformed Equation:

$$-\mu_n^2 \bar{\bar{U}}_{n,m} - \mu_m^2 \bar{\bar{U}}_{n,m} + \bar{\bar{S}}_{n,m}(t) = \frac{1}{w^2}\left(\frac{\partial^2 \bar{\bar{U}}_{n,m}}{\partial t^2} + 2\gamma \frac{\partial \bar{\bar{U}}_{n,m}}{\partial t}\right)$$

where $\bar{\bar{S}}_{n,m}(t) = \int_0^M \bar{S}_n(z,t) X_m(z)\, dz$

Laplace Transform - t

Transformed Equation: (with $u = u' = 0$ @ $t = 0$)

$$-\mu_n^2 w^2 \hat{U}_{n,m}(s) - \mu_m^2 w^2 \hat{U}_{n,m}(s) + \hat{\bar{\bar{S}}}_{n,m}(s) = s^2 \hat{U}_{n,m}(s) + 2\gamma s \hat{U}_{n,m}(s)$$

$$\hat{U}_{n,m}(s)(s^2 + 2\gamma s) + \hat{U}_{n,m}(s) w^2(\mu_n^2 + \mu_m^2) = \hat{\bar{\bar{S}}}_{n,m}(s)$$

$$\hat{U}_{n,m}(s) = \frac{\hat{\bar{\bar{S}}}_{n,m}(s)\, w^2}{s^2 + 2\gamma s + \gamma^2 + w^2(\mu_n^2 + \mu_m^2) - \gamma^2}$$

Substitute

$$R^2 = w^2(\mu_n^2 + \mu_m^2) - \gamma^2$$

$$= \hat{\bar{\bar{S}}}_{n,m}(s) \frac{w^2}{(s+\gamma)^2 + R^2}$$

$$= L\{\bar{\bar{S}}_{n,m}(t)\} \cdot L\left\{\frac{w^2 e^{-\gamma t} \sin(Rt)}{R}\right\}$$

Inverse Laplace Transform - t

$$\hat{U}_{n,m}(s) = L\{\bar{\bar{S}}_{n,m}(t)\}\, L\left\{\frac{W^2 e^{-\gamma t}\sin(Rt)}{R}\right\}$$

$$= L\left\{\bar{\bar{S}}_{n,m}(t) * \frac{W^2 e^{-\gamma t}\sin(Rt)}{R}\right\}$$

$$\bar{\bar{U}}_{n,m} = L^{-1}\left\{L\left\{\bar{\bar{S}}_{n,m}(t) * \frac{W^2 e^{-\gamma t}\sin(Rt)}{R}\right\}\right\}$$

$$\bar{\bar{U}}_{n,m}(t) = \bar{\bar{S}}_{n,m}(t) * \frac{W^2 e^{-\gamma t}\sin(Rt)}{R}$$

$$= \int_0^t \frac{W^2 e^{-\gamma(t-v)}\sin(R(t-v))}{R} \cdot \bar{\bar{S}}_{n,m}(v)\,dv$$

Inverse Finite Integral Transform - z

Inverse Transform: $\bar{U}_n(z,t) = \sum_{m=1}^{\infty} \bar{\bar{U}}_{n,m} \dfrac{X_m(z)}{\|X_m\|^2}$

$$\bar{U}_n(z,t) = \sum_{m=1}^{\infty} \int_0^t \frac{W^2 e^{-\gamma(t-v)}\sin(R(t-v))}{R} \cdot \bar{\bar{S}}_{n,m}(v)\,dv \,\frac{\sin(\frac{m\pi}{L}z)}{L/2}$$

Inverse Finite Integral Transform - x

$$U(x,z,t) = \sum_{n=1}^{\infty} \bar{U}_n \frac{U_n(x)}{\|U_n(x)\|^2_p}$$

$$U(x,z,t) =$$

$$\sum_{n=1}^{\infty}\left\{\sum_{m=1}^{\infty}\left[\int_0^t \frac{W^2 e^{-\gamma(t-v)}\sin(R(t-v))}{R} \cdot \bar{\bar{S}}_{n,m}(v)\,dv \,\frac{\sin(\frac{m\pi}{L}z)}{L/2}\right] \frac{x\left[J_0(\mu_n)Y_0(\mu_n x) - J_0(\mu_n x)Y_0(\mu_n)\right]}{\int_1^4 \frac{U_n^2(x)}{x}\,dx}\right\}$$

(7) $S(x,z,t) =$

$$\tilde{S}_n(z,t) = \int_1^4 S_0\,\delta(x-x_0)\,\delta(z-z_0)\,\delta(t-t_0)\,U_n(x)\,p(x)\,dx$$

$$= S_0\,\delta(z-z_0)\,\delta(t-t_0)\,U_n(x_0)\,p(x_0)$$

$$\tilde{\tilde{S}}_{n,m}(t) = \int_0^M \tilde{S}_n(z,t)\,X_m(z)\,dz$$

$$= \int_0^M S_0\,\delta(z-z_0)\,\delta(t-t_0)\,U_n(x_0)\,p(x_0)\,X_m(z)\,dz$$

$$= S_0\,\delta(t-t_0)\,U_n(x_0)\,p(x_0)\,X_m(z_0)$$

$u(x,z,t) =$

$$\sum_{n=1}^{\infty}\left\{\sum_{m=1}^{\infty}\left[\underbrace{\int_0^t \frac{w^2 e^{-\gamma(t-v)}\sin(R(t-v))}{R}\cdot\tilde{\tilde{S}}_{n,m}(v)\,dv}\ \frac{\sin(\frac{m\pi}{L}z)}{L/2}\right]\frac{x\left[J_0(\mu_n)Y_0(\mu_n x)-J_0(\mu_n x)Y_0(\mu_n)\right]}{\int_1^4 \frac{U_n^2(x)}{x}\,dx}\right\}$$

$$= \underbrace{\frac{w^2 U_n(x_0)\,p(x_0)\,X_m(z_0)\,S_0}{R}}_{\text{call this } A}\int_0^t e^{-\gamma(t-v)}\sin(R(t-v))\,\delta(t-t_0)\,dv$$

with $t_0 = 0.5$

$$= A e^{-\gamma(t-0.5)}\left(2\overset{\nearrow 1}{H(t)}-1\right)H\left(t\overset{\nearrow 1}{H(t)}-0.5\right)\overset{\nearrow 1}{H}\left(0.5-t\overset{\nearrow 0}{H(-t)}\right)\sin(R(t-0.5))$$

$$= A e^{\gamma(t_0-t)}H(t-t_0)\sin(R(t-t_0))$$

Solution:

$$u(x,z,t) = \sum_{n=1}^{\infty}\left\{\sum_{m=1}^{\infty}\left[A e^{\gamma(t_0-t)}H(t-t_0)\sin(R(t-t_0))\,\frac{\sin(\frac{m\pi}{L}z)}{L/2}\right]\frac{x\left[J_0(\mu_n)Y_0(\mu_n x)-J_0(\mu_n x)Y_0(\mu_n)\right]}{\int_1^4 \frac{U_n^2(x)}{x}\,dx}\right\}$$

where $A = \dfrac{w^2 U_n(x_0)\,p(x_0)\,X_m(z_0)\,S_0}{R}$, $R = \left(w^2(\mu_n^2+\mu_m^2)-\gamma^2\right)^{1/2}$

and all other pieces are defined previously in parts 1-6

(7/8) $x_0 = 3$

$x_1 = 1$

$x_2 = 4$

$M = 2$

$z_0 = 1$

$S_0 = 0.5$

$w = 0.5$

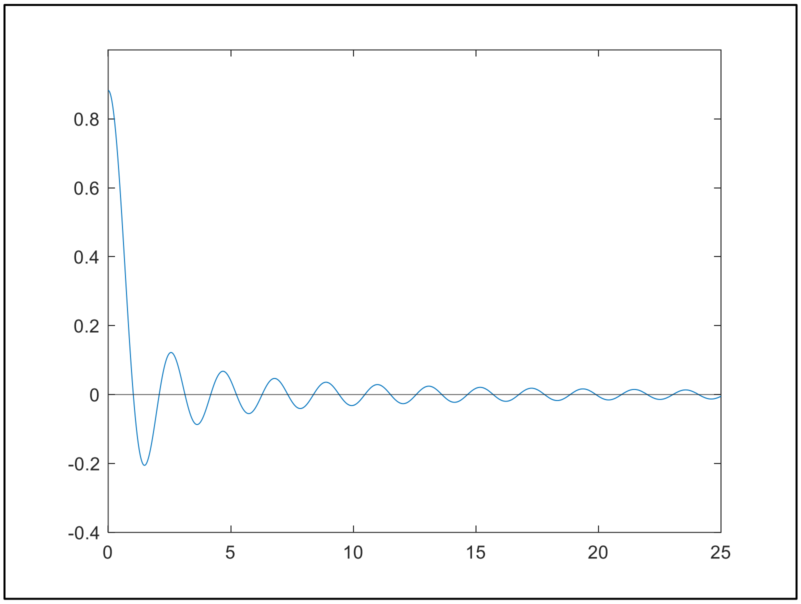
$t_0 = 0.5$

$\gamma = 0.9$

* See plots / code attatched to the end of this pdf

Part 2

Solving for eigenvalues:



Editor - meen505FINAL.m

+6 | meen510_Project.m | obliqueshock.m | thetabetaMrel.m | flowprandtlmeyer.m | meen512hw10_problem5.m | meen505FINAL.m | besselthingy.m

Variables - u

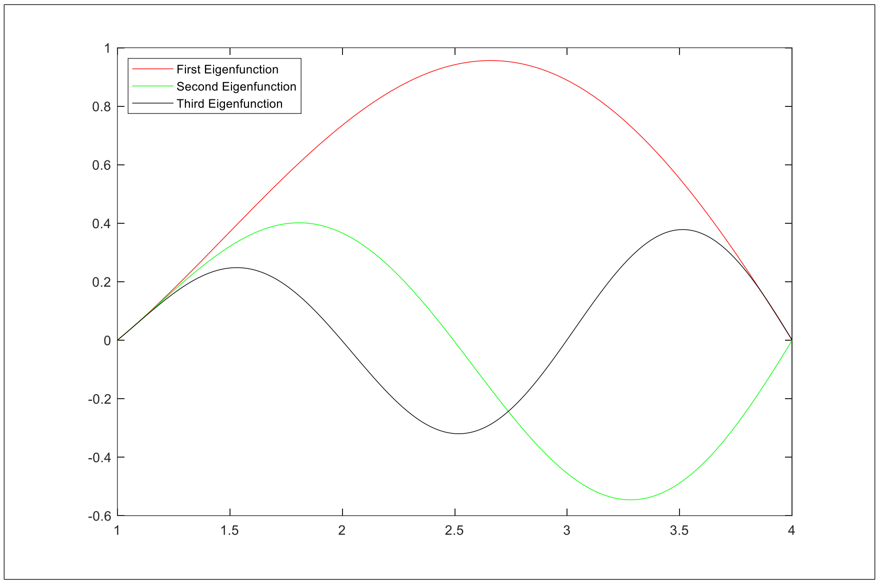
```
clc; clear; close all;
xmax = 25;


x = [0:0.001:xmax];
y = besselj(0,x).*bessely(0,4*x)-besselj(0,4*x).*bessely(0,x) ;


% syms u
% eqn = besselj(0,u).*bessely(0,4*u)-besselj(0,4*u).*bessely(0,u) == 0;
% vpasolve(eqn,u)


plot(x,y)
hold on
yline(0)

f = @besselthingy;

xval = fzeros(f,0,xmax)

```

Command Window

New to MATLAB? See resources for Getting Started.

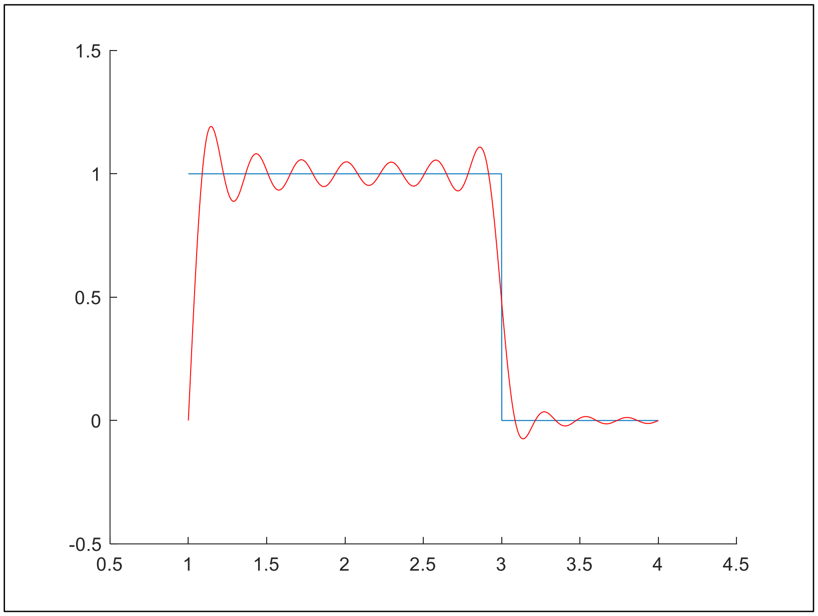
```
xval =

  Columns 1 through 13

    1.0244    2.0809    3.1322    4.1816    5.2301    6.2783    7.3262    8.3739    9.4215   10.4690   11.5165   12.5639   13.6113

  Columns 14 through 23

   14.6586   15.7060   16.7533   17.8006   18.8479   19.8952   20.9425   21.9897   23.0370   24.0842

fx >>
```

Plotting Eigenfunctions




```
x1 = [1:0.001:4];

y1 = x1.*(besselj(0,u_n(1)).*bessely(0,u_n(1).*x1)-besselj(0,u_n(1).*x1).*bessely(0,u_n(1)));
y2 = x1.*(besselj(0,u_n(2)).*bessely(0,u_n(2).*x1)-besselj(0,u_n(2).*x1).*bessely(0,u_n(2)));
y3 = x1.*(besselj(0,u_n(3)).*bessely(0,u_n(3).*x1)-besselj(0,u_n(3).*x1).*bessely(0,u_n(3)));


figure
plot(x1,y1,'r','DisplayName','First Eigenfunction')
hold on
plot(x1,y2,'g','DisplayName','Second Eigenfunction')
hold on
plot(x1,y3,'k','DisplayName','Third Eigenfunction')
legend('location','NorthWest')
```

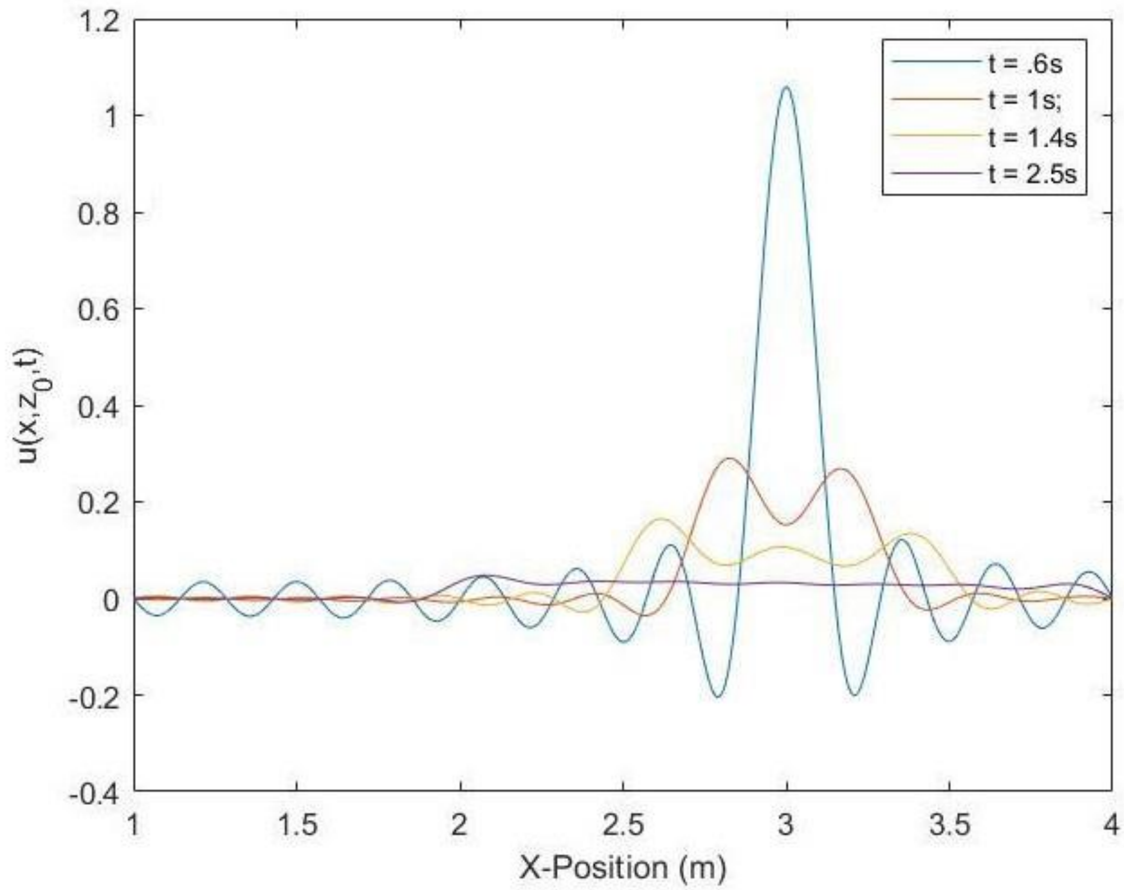
Generalized Fourier Series of: f(x) = 1 – H(x-3)



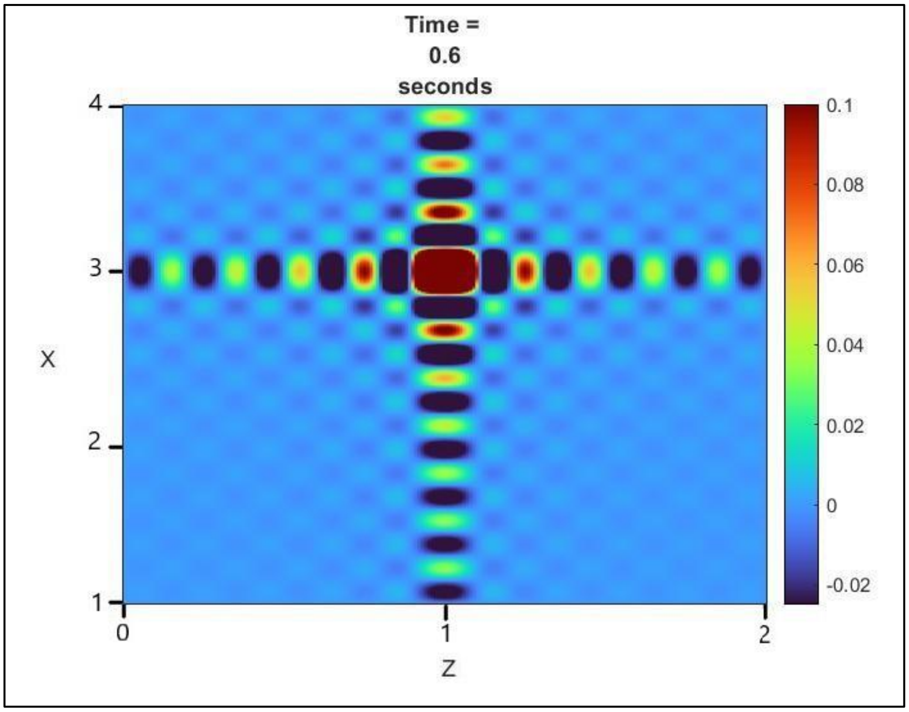

```
fourier_series_x = x1.*0;
% q_an_running_total = 0;
% q_norm_running_total = 0;

for i = 1:20

fun_an = @(x) (1-heaviside(x-3)).*(1./x).*x.*((besselj(0,u_n(i)).*bessely(0,u_n(i).*x)-besselj(0,u_n(i).*x).*bessely(0,u_n(i))));

q_an = integral(fun_an,1,4)

%q_an_running_total = q_an_running_total + q_an;

fun_norm = @(x) (1./x).*((x).*(besselj(0,u_n(i)).*bessely(0,u_n(i).*x)-besselj(0,u_n(i).*x).*bessely(0,u_n(i)))).^2;

q_norm = integral(fun_norm,1,4)

%q_norm_running_total = q_norm_running_total + q_norm;

% fourier_series_x = fourier_series_x + (q_an/q_norm).*x1.*exp(u_n(i).*x1).*(besselj(0,u_n(i)).*bessely(0,u_n(i).*x1)-besselj(0,u_
fourier_series_x = fourier_series_x + (q_an/q_norm).*x1.*(besselj(0,u_n(i)).*bessely(0,u_n(i).*x1)-besselj(0,u_n(i).*x1).*bessely(

end
%% Plot Heaviside Function
heaviside_function = 1 - heaviside(x1-3);

figure
hold on
plot(x1,heaviside_function)
xlim([0.5,4.5])
ylim([-0.5,1.5])
plot(x1,fourier_series_x,'r')
```

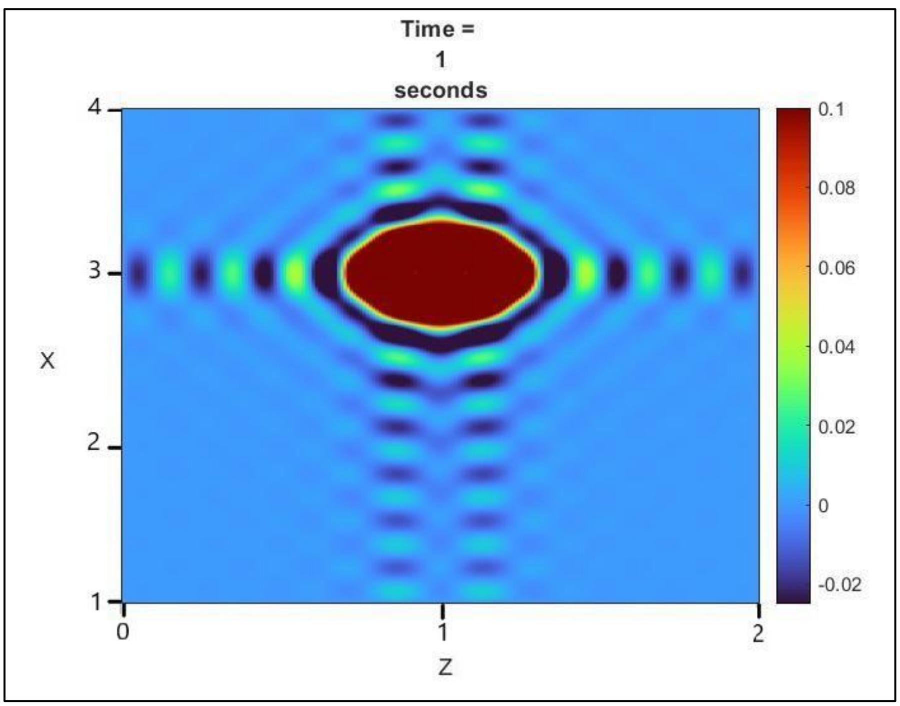
Part 7/8

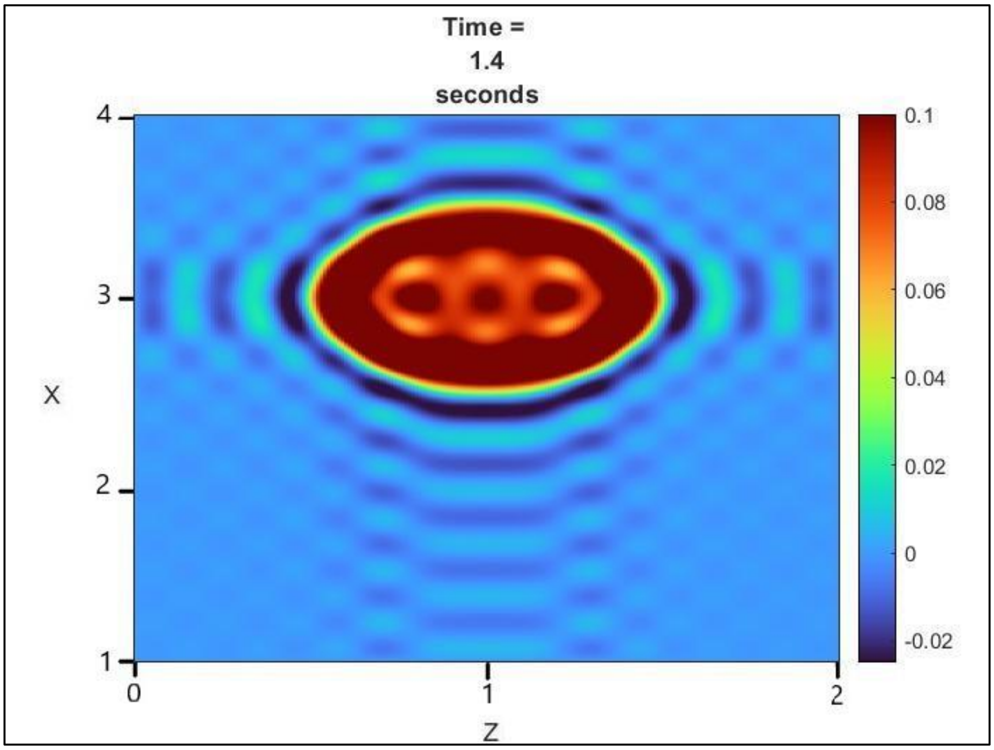
Line plot at z = z0

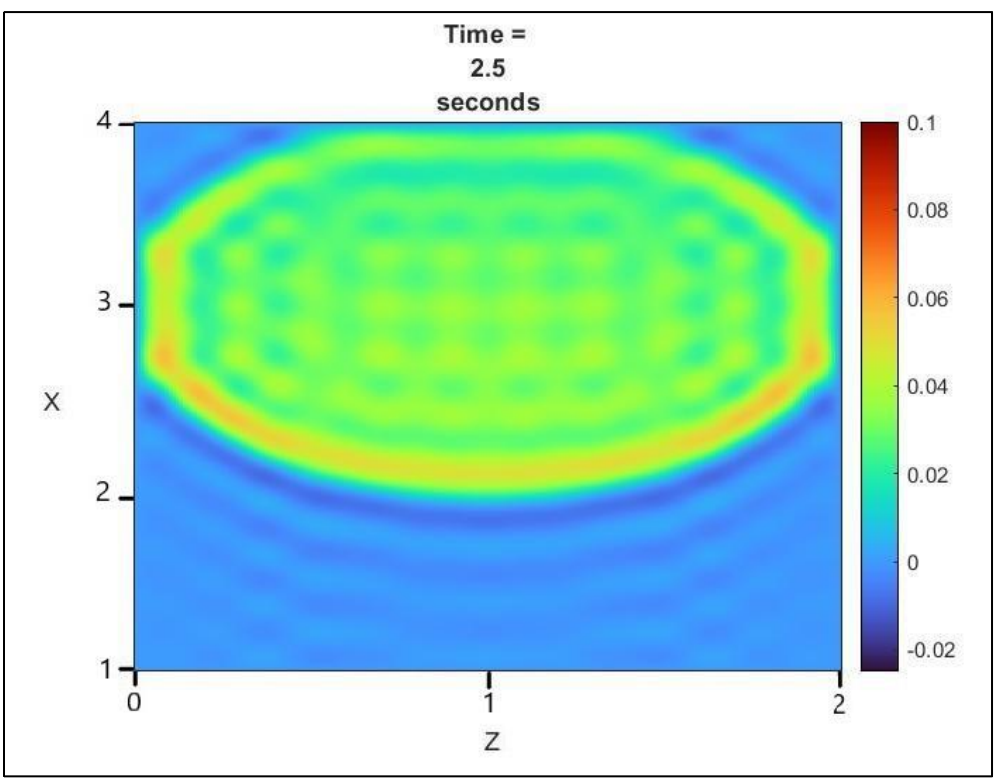


Solution curves for given time stamps (colored bar on the right is the value of u(x,z,t):






Time =
1.4
seconds
4
3
2
1
X
0
1
2
Z
0.1
0.08
0.06
0.04
0.02
0
-0.02


Time =
2.5
seconds
4
3
2
1
X
0
1
2
Z
0.1
0.08
0.06
0.04
0.02
0
-0.02

Contents

use eigenfunctions to appx 1-H(x-3)

```
x = 1:.01:4;
mu_n = [1.02442133426666 2.08094334602356       3.13216686248779        4.18156909942627        5.23014783859253        6.27828788757324        7.32616853713989
p = 1./x;
for i = 1:length(mu_n)

    Xn(i,:) = x.*(besselj(0,4*mu_n(i)).*bessely(0,mu_n(i).*x)-besselj(0,mu_n(i).*x)*bessely(0,4*mu_n(i)));
    wip(i,:) = Xn(i,:).^2.*p;
    Xn_sqp(i) = trapz(x,wip(i,:));
    inner_An(i,:) = Xn(i,:).*(1-heaviside(x-3)).*p;
    An(i) = trapz(x,inner_An(i,:));%/Xn_sqp(i);
    %An(i) = trapz(x,(Xn(i,:).*(1-heaviside(x-3))).*p(i))/Xn_sqp(i);
end


sum = 0;
for k = 1:length(x)
    for j = 1:length(mu_n)
        value = An(j).*Xn(j,k)/Xn_sqp(j);
        sum = sum + value;

    end

    g(k) = sum;
    sum = 0;
end

plot(x,1-heaviside(x-3))
hold on
plot(x,g)
%plot(x,inner_An)
```

part 6 and 7

```
M = 2;
x0 = 3;
x1 = 1;
x2 = 4;
z0 = 1;
S0 = .5;
w = .5;
t0 = .5;
g = .9;
z = 0:.01:M;
t = [.6 1.0 1.4 2.5];

for m = 1:20
    mu_m(m) = m*pi/M;
    Zm(m,:) = sin(mu_m(m).*z);
end

for m = 1:20
    for n = 1:20
        Bnm(n,m) = sin(mu_m(m)*z0)*(besselj(0,mu_n(n))*bessely(0,mu_n(n)*x0)-besselj(0,mu_n(n)*x0)*bessely(0,mu_n(n)));
        Rnm(n,m) = sqrt(w^2*(mu_n(n)^2+mu_m(m)^2)-g^2);
    end
end

t1 = t(4);
t = 0:.1:6;
summ = 0;
sumn = 0;
for q = 1:length(t)
    t1 = t(q);
for i = 1:length(x)
    for k = 1:length(z)
        for n = 1:20
            for m = 1:20
                constant = w^2*S0/Rnm(n,m);

                val = constant*Bnm(n,m)*exp(g*(t0-t1))*heaviside(t1-t0)*sin(Rnm(n,m)*(t1-t0));
                val2 = val*Zm(m,k)*2/M;
                summ = summ+val2;
            end
            valx = summ*Xn(n,i)/Xn_sqp(n);
            sumn = valx+sumn;
            summ = 0;
        end
        u_zxt(i,k,q) = sumn;
```

```
        sumn = 0;

    end
end
end



%
% surf(z,x,u_zxt)
% figure
% heatmap(z,x,u_zxt)
```

plotting

```
myVideo = VideoWriter('Me_505-Final'); %open video file
myVideo.FrameRate = 10;  %can adjust this, 5 - 10 works well for me
open(myVideo)

for j = 1:length(u_zxt(1,1,:))
    heatmap(z,x,u_zxt(:,:,j))
    Ax = gca;
    Ax.XDisplayLabels = nan(size(Ax.XDisplayData));
    Ax.YDisplayLabels = nan(size(Ax.YDisplayData));
    caxis([-.1 .025])
    colormap('turbo')

    title({'Time = ' t(j) 'seconds'})

    xlabel('Z')
    ylabel('X')
    %clabel('u(x,z,t)')
    grid off
    pause(.1)

    frame = getframe(gcf); %get frame
    writeVideo(myVideo, frame);
end
close(myVideo)
```

make a gif

```
i = 26
%plot first frame
    h = heatmap(z,x,u_zxt(:,:,i))
    Ax = gca;
    Ax.XDisplayLabels = nan(size(Ax.XDisplayData));
    Ax.YDisplayLabels = nan(size(Ax.YDisplayData));
    caxis([-.1 .025])
    colormap('turbo')

    title({'Time = ' t(i) 'seconds'})

    xlabel('Z')
    ylabel('X')
    %clabel('u(x,z,t)')
    grid off
gif('Me_505_Final.gif')

for k = 2:length(t)
    h = heatmap(z,x,u_zxt(:,:,k))
    Ax = gca;
    Ax.XDisplayLabels = nan(size(Ax.XDisplayData));
    Ax.YDisplayLabels = nan(size(Ax.YDisplayData));
    caxis([-.1 .025])
    colormap('turbo')

    title({'Time = ' t(k) 'seconds'})

    xlabel('Z')
    ylabel('X')
    grid off
   gif
end
```