ME-505 FALL 2021 **FINAL EXAM**

You can work in teams of two students.
However, you should turn in your own test, indicating the name of your teammate.

NAME:

Teammate (if any):

1. Submit solutions through the Learning Suite (**single file**) before Midnight on **Friday, December 17.**
2. *Please write neatly and show all your work*.
3. Any content of the exam cannot be discussed with anyone except of your teammate.
4. Computer and standard math software can be used only as the supportive tools.
5. Class notes, class web-site, and regular math books can be used.
6. No set of rules can cover all possible situations – be reasonable – do only what you believe is proper.

SIBRA

Consider the following initial boundary value problem for the function $u(x,z,t)$:

$$\frac{\partial^2 u}{\partial x^2}-\frac{1}{x}\frac{\partial u}{\partial x}+\frac{1}{x^2}u+\frac{\partial^2 u}{\partial z^2}+S(x,z,t)=\frac{1}{w^2}\left(\frac{\partial^2 u}{\partial t^2}+2\gamma\frac{\partial u}{\partial t}\right),\quad x_1<x<x_2,\quad 0<z<M,\quad t>0$$

$$u(x_1,z,t)=0 \qquad 0<z<M, \qquad t>0$$

$$u(x_2,z,t)=0 \qquad 0<z<M, \qquad t>0$$

$$u(x,0,t)=0 \qquad x_1<x<x_2, \qquad t>0$$

$$u(x,M,t)=0 \qquad x_1<x<x_2, \qquad t>0$$

$$u(x,z,0)=0 \qquad x_1\le x\le x_2, \qquad 0\le z\le M$$

$$\frac{\partial}{\partial t}u(x,z,0)=0 \qquad x_1\le x\le x_2, \qquad 0\le z\le M$$

1) Follow the Outline of the Finite Integral Transform method (Section IX.5.5, p.862).

Analyze the differential operator *L* with respect to variable x.

Rewrite operator *L* in self-adjoint form and find the weight function $p(x)$.

2) Set the supplemental Eigenvalue Problem for operator *L*.

Use the Sturm-Liouville Theorem to find the general solution of the differential equation

(*hint:* you may consider the equations solvable in terms of Bessel functions).

Apply the boundary conditions to generate eigenvalues and corresponding eigenfunctions.

Write the first three eigenvalues and sketch the first three eigenfunctions for $x_1=1.0,\ x_2=4.0$.

Define the weighted inner product.

Define the square of the norm of eigenfunctions (do not calculate in closed form).

3) Use the found eigenfunctions to represent the step-wise constant function $f(x)=1-H(x-3)$ by the Generalized Fourier series in the interval $x_1\le x\le x_2$. Sketch the graph of $f(x)$ and truncated Fourier series (with 20 terms).

4) Define the Finite Integral Transform pair based on the found eigenfunctions.

5) Derive the operational property of the defined integral transform: apply transform to the differential operator L .

6) Use the defined Finite Integral Transform in *x* variable, the Finite Integral Transform in z variable, and the Laplace transform to solve the given i.b.v.p.

Write the formal solution for the transformed function $\bar{\bar{u}}_{n,m}(t)$ using the convolution theorem.

Use the inverse Finite Integral Transforms to write the solution for $u(x,z,t)$.

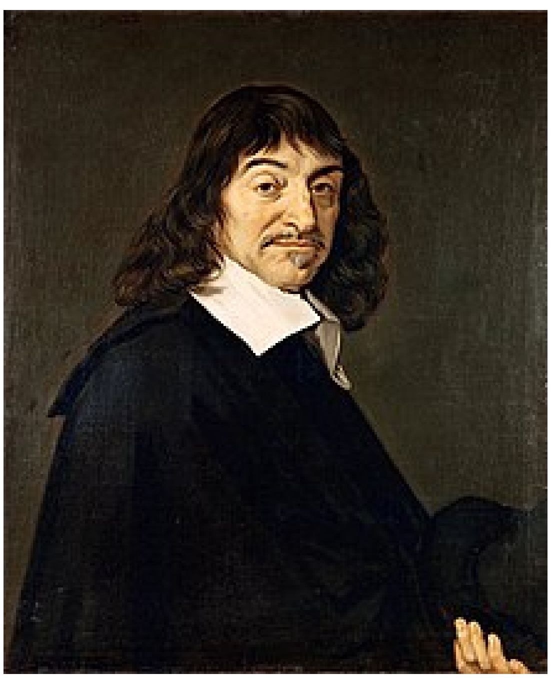
7) Find the solution for the case

$$S(x,z,t)=S_0\delta(x-x_0)\delta(z-z_0)\delta(t-t_0),\ f_1(t)=0,\ f_2(t)=0,\ u(x,z,0)=0,\ \frac{\partial}{\partial t}u(x,z,0)=0$$

$$x_0=3.0, x_1=1.0,\ x_2=4.0,\ M=2.0,\ z_0=1.0,\ S_0=0.5,\ w=0.5,\ t_0=0.5,\ \gamma=0.9$$

8) Sketch the solution curves for $u(x,\ z_0,\ t)$ for the moments of time $t=\ 0.6,\ 1.0,\ 1.4,\ 2.5$.

Philippe de Champaigne   Portrait of two men



Portrait of René Descartes

after Frans Hals

Portrait of Blaise Pascal

Philippe de Champaigne