ENG T 502 · ADVANCED APPLIED ENGINEERING MATHEMATICS I

Final Exam (Part I)

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Name: ______________________	Section: 1

1. (a) Show that any function $f : (-\pi, \pi) \to \mathbb{R}$ can be written uniquely as the sum of an even function and an odd function, called its *even* and *odd components.*

   (b) Write the function

$$f(x) := \begin{cases} 0, & -\pi < x \leq 0 \\ \sin x, & 0 < x < \pi \end{cases}$$

   as the sum of its even and ood components, and then find the Fourier series on $(-pi, \pi)$.

2. (a) Show that the Fourier series of the function given by $f(x) := |\sin x|$ on $(-\pi, \pi)$ converges to $f(x)$ at each point x in $[-\pi, \pi]$.

   (b) Establish the identities

      i. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} = \frac{1}{2}$

      ii. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4n^2 - 1} = \frac{\pi - 2}{4}$

   by substituting suitable values of x into the Fourier expansion of f.

3. (a) Find the Eigenvalues and Eigenfunctions of the following Sturm-Liouville problem

$$\begin{aligned} y'' + y' + (1 + \lambda)y &= 0 \\ y(0) &= 0 \\ y(1) &= 0 \end{aligned}$$

   (b) Use the Eigenfunctions obtained in (a) to derive an Eigenfunction expansion of the function given by

$$f(x) = xe^{-x}$$

   on the interval $[0, 1]$.

Please submit the solutions to problems 1 and 2 to *Reinhard Franz* and the solution to problem 3 to *Vladimir Solovjov*!