Due Friday June 17,   8am   (under the door of the room CB 133)

1.   a)   Reduce the following BVP to a Sturm-Liouville problem:

$$u'' - 2au' + b^2 \lambda u = 0 \qquad\qquad a, b \in \mathbf{R}$$

$$u(0) = 0$$

$$u'(L) + Hu(L) = 0$$

and find the eigenvalues and the eigenfunctions.

b)   Use the obtained set of eigenfunctions for generalized Fourier series representation of the function

$$f(x) = xe^{-x} \qquad \text{in the interval } (0, L)$$

Sketch the graph for $n = 6$ and $n = 15$.   a=2, b=1, H=0.5, L=0.2

(Do not forget about the weight function in the inner product!)

2.   a)   Find the solution of the Heat Equation $u(x, y, t)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + F(x, y) = a^2 \frac{\partial u}{\partial t} \qquad t > 0$$

in the domain $D = (0, L) \times (0, M)$ subject to boundary conditions:

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = 0, \quad u|_{x=L} = 0, \quad \left.-\frac{\partial u}{\partial y} + Hu\right|_{y=0} = 0, \quad u|_{y=M} = f(x) \qquad t > 0$$

and initial condition:

$$u(x, y, 0) = u_0(x, y)$$

b)   Consider:
$$L = 4, \ M = 2, \ a = 50, \ H = 3$$
$$f(x) = 5$$
$$u_0(x) = x(L - x)y(M - y)$$
$$F(x, y) = -2$$

c)   What is $u(x = 1, y = 1, t = 15) = ?$

Comments for those who use Maple:
1. Use just *4-5* terms in the truncated series solution.

2. If double integration is difficult for Maple use numerical integration.

3. Maple performs integration with some methods which yield small imaginary residue. In this case function will not be plotted. To resolve this problem use only the real part of the result.

For example, in calculation of the Fourier coefficients by double integration:

```
> B[n,m]:=Re(eval(Int(Int(U0(x,y)*X[n]*Y[m],x=0..L),y=0..M)/NX[n]/NY[m]));
```

(note that the integration command is capitalized)