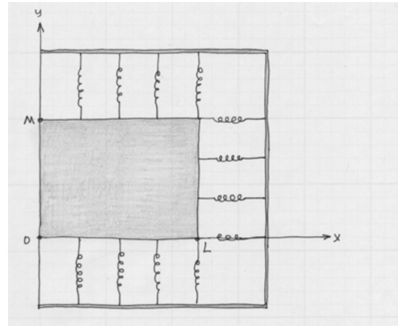
1.



Consider the IBVP describing vibration of the rectangular membrane with one fixed side and with other sides attached by the springs the frame.
Use the Integral Transform Methods to solve this problem.

a) Find the solution of the Wave Equation $u(x,y,t)$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+g(x,y,t)=a^2\frac{\partial^2 u}{\partial t^2}+b^2\frac{\partial u}{\partial t}+c^2 u \qquad t>0 \qquad (*)$$

in the domain $D=(0,L)\times(0,M)$ subject to boundary conditions:

$x=0 \quad u\big|_{x=0}=f_3(y,t) \qquad t>0 \qquad H_1=\dfrac{h_1}{k}$ (Dirichlet)

$x=L \quad \left[k\dfrac{\partial u}{\partial y}+h_4 u\right]\Bigg|_{x=L}=f_4(y,t) \qquad t>0 \qquad H_2=\dfrac{h_2}{k}$ (Robin)

$y=0 \quad \left[-k\dfrac{\partial u}{\partial y}+h_1 u\right]\Bigg|_{y=0}=f_1(x,t) \qquad t>0 \qquad H_3=\dfrac{h_3}{k}$ (Robin)

$y=M \quad \left[k\dfrac{\partial u}{\partial y}+h_2 u\right]\Bigg|_{y=M}=f_2(x,t) \qquad t>0 \qquad H_4=\dfrac{h_4}{k}$ (Robin)

and initial conditions:

$u(x,y,0)=u_0(x,y)$ (initial shape of the membrane)

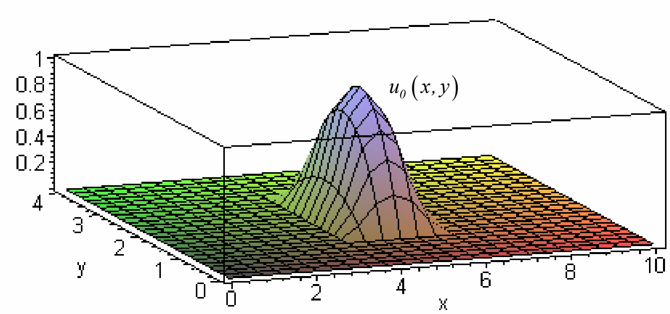
$\dfrac{\partial u(x,y,0)}{\partial t}=u_1(x,y)$ (initial velocity of the membrane)

b) Write the solution and sketch the graph for the case:

$L=10,\ M=4,$ $\qquad g(x,y,t)=0$

$a=1.0,\ b=0.8,\ c=1.0$ $\qquad f_1=f_2=f_3=f_4=0$

$k=2,\ h_1=h_2=h_3=h_4=4.0$



$u_0(x,y)=(x-4)(6-x)(y-1)(3-y)\left[H(x-4)-H(x-6)\right]\left[H(y-1)-H(y-3)\right]$

$u_1(x,y)=0$

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c) Investigate the influence of coefficients *a, b* and *c* on the vibration of the membrane.