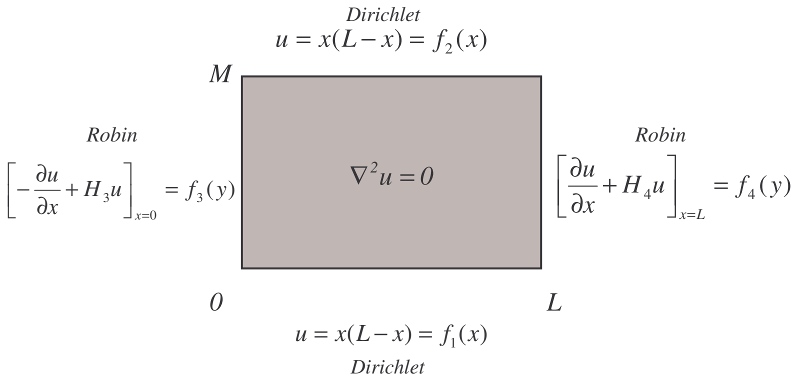
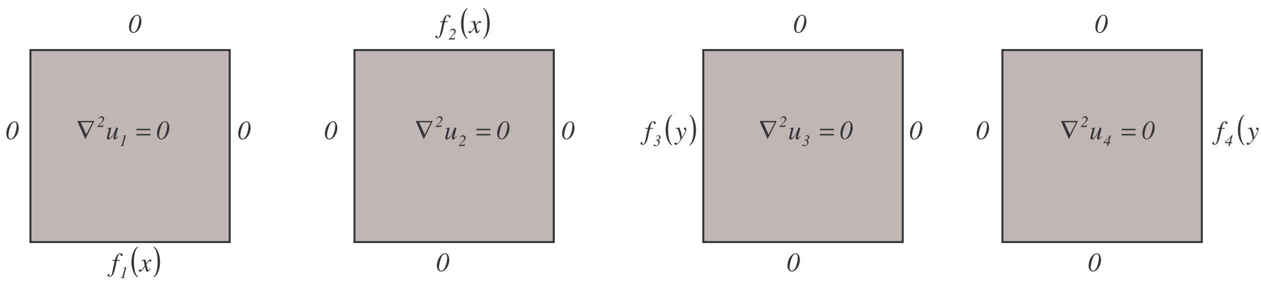
The Laplace Equation: 09 – DDRR (Dirichlet- Dirichlet-Robin-Robin)



Supplemental problems: ↓



Solution of supplemental problems:

$$u_1(x,y) = \sum_{n=1}^{\infty} a_n \sinh[\lambda_n (y-M)](\lambda_n \cos\lambda_n x + H_3 \sin\lambda_n x)$$

$$a_n = \frac{\int_0^L x(L-x)(\lambda_n \cos(\lambda_n x) + H_3 \sin(\lambda_n x))dx}{-\sinh(\lambda_n M)\left[\left(\frac{\lambda_n^2 + H_3^2}{2}\right)\left(L + \frac{H_4}{\lambda_n^2 + H_2^2}\right) + \frac{H_3}{2}\right]}$$

$$u_2(x,y) = \sum_{n=1}^{\infty} b_n \sinh(\lambda_n y)(\lambda_n \cos\lambda_n x + H_3 \sin\lambda_n x)$$

$$b_n = \frac{\int_0^L x(L-x)(\lambda_n \cos\lambda_n x + H_3 \sin\lambda_n x)dx}{\sinh\lambda_n M\left[\left(\frac{\lambda_n^2 + H_3^2}{2}\right)\left(L + \frac{H_4}{\lambda_n^2 + H_2^2}\right) + \frac{H_3}{2}\right]}$$

Where λ_n from positive roots of:

$$\left(H_3 H_4 - \lambda^2\right)\sin\lambda L + (H_3 + H_4)\lambda\cos\lambda L = 0$$

$$u_3(x,y) = \sum_{n=1}^{\infty} c_n\left[\cosh\left[\frac{n\pi}{M}(x-L)\right] - \frac{H_4 M}{n\pi}\sinh\left[\frac{n\pi}{M}(x-L)\right]\right]\sin\left(\frac{n\pi}{M}y\right)$$

$$c_n = \frac{\frac{6}{M}\int_0^M \sin\left(\frac{n\pi}{M}y\right)dy}{\cosh\left(\frac{n\pi}{M}L\right) + \frac{H_4 M}{n\pi}\sinh\left(\frac{n\pi}{M}L\right)}$$

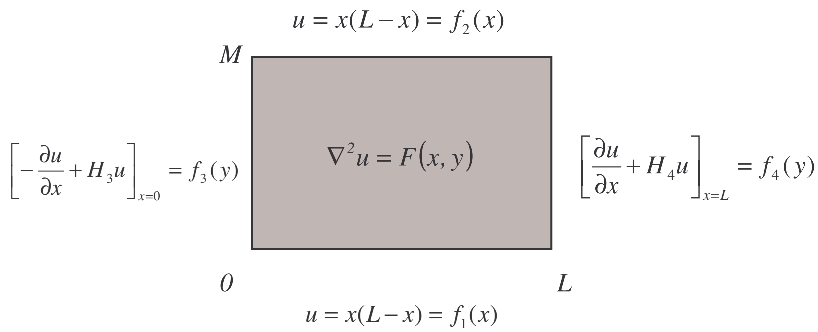
$$u_4(x,y) = \sum_{n=1}^{\infty} d_n\left[\cosh\frac{n\pi}{M}x + \frac{H_3 M}{n\pi}\sinh\frac{n\pi}{M}x\right]\sin\frac{n\pi}{M}y$$

$$d_n = \frac{\frac{8}{M}\int_0^M \sin\frac{n\pi}{M}y\,dy}{\cosh\frac{n\pi}{M}L + \frac{H_3 M}{n\pi}\sinh\frac{n\pi}{M}L}$$

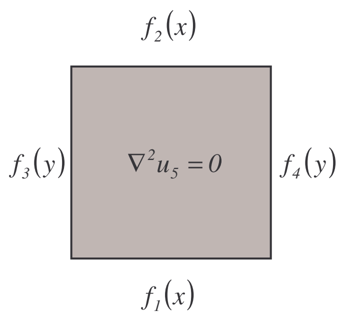
Solution of BVP problem:

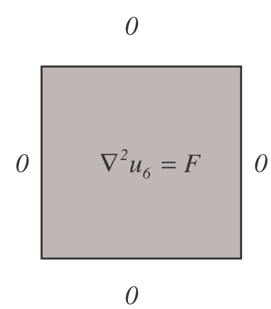
$$u(x,y) = u_1(x,y) + u_2(x,y) + u_3(x,y) + u_4(x,y)$$

POISSON'S EQUATION: 01 – DDDD



Supplemental problems: ↓





Solution of supplemental problems:

Solution of Laplace's homogeneous equation with non-homogeneous b.c.'s:

$$u_5(x, y) = u_1(x, y) + u_2(x, y) + u_3(x, y) + u_4(x, y)$$

Solution of Poisson's equation with homogeneous boundary conditions

$$u_6 = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{nm}(\lambda_n \cos\lambda_n x + H_3 \sin\lambda_n x)\sin\left(\frac{m\pi}{M}y\right)$$

$$A_{nm} = \frac{-\int_0^L\int_0^M F(x, y)(\lambda_n \cos\lambda_n x + H_3 \sin\lambda_n x)\sin\left(\frac{m\pi}{M}y\right)dxdy}{\frac{M}{2}\left[\left(\frac{\lambda_n^2 + H_3^2}{2}\right)\left(L + \frac{H_4}{\lambda_n^2 + H_4^2}\right) + \frac{H_3}{2}\right]\left(\lambda_n^2 + \frac{m^2\pi^2}{M^2}\right)}$$

Where λ_n from positive roots of: $(H_3 H_4 - \lambda^2)\sin\lambda L + (H_3 + H_4)\lambda\cos\lambda L = 0$

Solution of BVP for Poisson's Equation (superposition principle):

$$u(x, y) = u_5(x, y) + u_6(x, y)$$