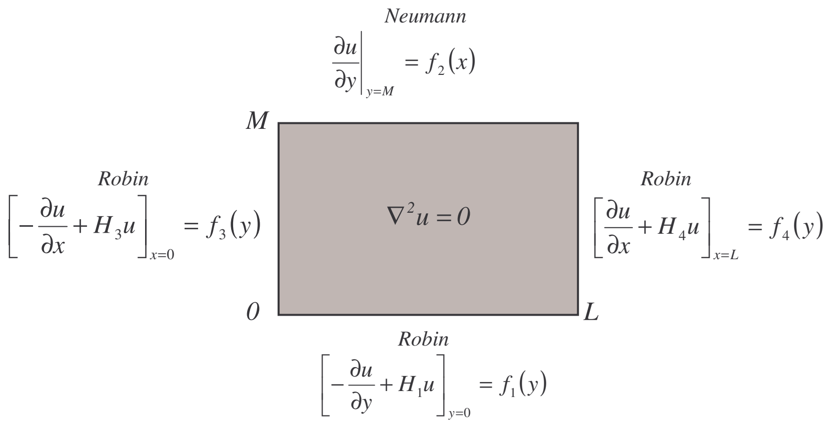
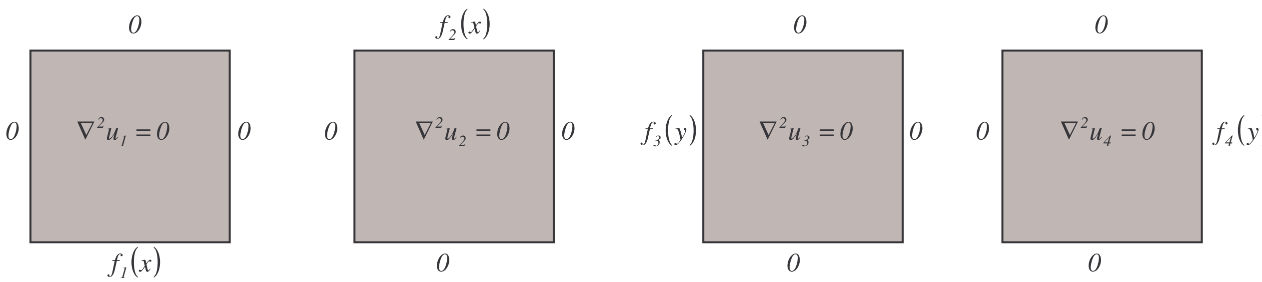
The Laplace Equation: 12 – RNRR (Robin-Neumann-Robin-Robin)



Supplemental problems: ↓



Solution of supplemental problems:

$$u_1(x,y) = \sum_{n=1}^{\infty} a_n \cosh[\lambda_n (y-M)][\lambda_n \cos(\lambda_n x) + H_3 \sin(\lambda_n x)]$$

$$a_n = \frac{\int_0^L f_1(x)[\lambda_n \cos(\lambda_n x) + H_3 \sin(\lambda_n x)]dx}{[\lambda_n \sinh(\lambda_n M) + H_1 \cosh(\lambda_n M)]\left[\frac{(\lambda_n^2 + H_3^2)}{2}\left(L + \frac{H_4}{\lambda_n^2 + H_4^2}\right) + \frac{H_3}{2}\right]}$$

$$u_2(x,y) = \sum_{n=1}^{\infty} b_n \left[\cosh(\lambda_n y) + \frac{H_1}{\lambda_n}\sinh(\lambda_n y)\right][\lambda_n \cos(\lambda_n x) + H_3 \sin(\lambda_n x)]$$

$$b_n = \frac{\int_0^L f_2(x)[\lambda_n \cos(\lambda_n x) + H_3 \sin(\lambda_n x)]dx}{[\lambda_n \sinh(\lambda_n M) + H_1 \cosh(\lambda_n M)]\left[\frac{(\lambda_n^2 + H_3^2)}{2}\left(L + \frac{H_4}{\lambda_n^2 + H_4^2}\right) + \frac{H_3}{2}\right]}$$

Where λ_n *are positive roots of*

$$(H_3 H_4 - \lambda^2)\sin(\lambda L) + (H_3 + H_4)\lambda \cos(\lambda L) = 0$$

$$u_3(x,y)=\sum_{n=1}^{\infty} c_n \cos[\lambda_n(y-M)]\left[\cosh[\lambda_n(x-L)]-\frac{H_4}{\lambda_n}\sinh[\lambda_n(x-L)]\right]$$

$$c_n=\frac{\int_0^M f_3(y)\cos[\lambda_n(y-M)]dy}{\left[\left(\lambda_n-\frac{H_3H_4}{\lambda_n}\right)\sinh(\lambda_n L)+(H_3+H_4)\cosh(\lambda_n L)\right]\left[\frac{M}{2}+\frac{\sin(2\lambda_n M)}{4\lambda_n}\right]}$$

$$u_4(x,y)=\sum_{n=1}^{\infty} d_n \cos[\lambda_n(y-M)]\left[\cosh(\lambda_n x)+\frac{H_3}{\lambda_n}\sinh(\lambda_n x)\right]$$

$$d_n=\frac{\int_0^M f_4(y)\cos[\lambda_n(y-M)]dy}{\left[\left(\lambda_n-\frac{H_3H_4}{\lambda_n}\right)\sinh(\lambda_n L)+(H_3+H_4)\cosh(\lambda_n L)\right]\left[\frac{M}{2}+\frac{\sin(2\lambda_n M)}{4\lambda_n}\right]}$$

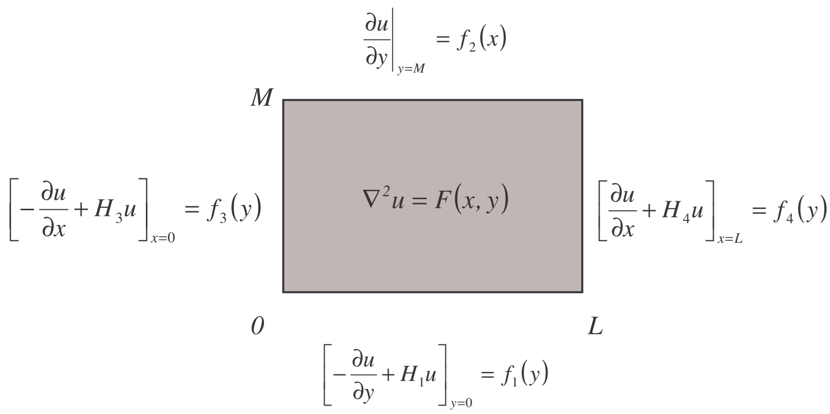
Where λ_n *are positive roots of*

$$\lambda\sin(\lambda M)-H_1\cos(\lambda M)=0$$

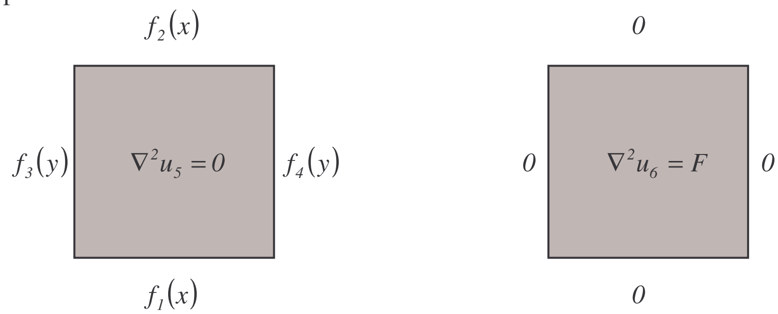
Solution of BVP problem:

$$u(x,y)=u_1(x,y)+u_2(x,y)+u_3(x,y)+u_4(x,y)$$

POISSON'S EQUATION: 12 – RNRR



Supplemental problems: ↓



<u>Solution of supplemental problems:</u>

Solution of Laplace's homogeneous equation with non-homogeneous b.c.'s:

$$u_5(x, y) = u_1(x, y) + u_2(x, y) + u_3(x, y) + u_4(x, y)$$

Solution of Poisson's equation with homogeneous boundary conditions

$$u_6(x, y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{n,m}\left[\lambda_n \cos(\lambda_n x) + H_3 \sin(\lambda_n x)\right]\cos\left[\mu_m (y - M)\right]$$

$$A_{n,m} = \frac{\int_0^L\int_0^M F(x, y)\left[\lambda_n \cos(\lambda_n x) + H_3 \sin(\lambda_n x)\right]\cos\left[\mu_m (y - M)\right]dxdy}{\left(\lambda_n^2 + \mu_m^2\right)\left[\frac{\left(\lambda_n^2 + H_3^2\right)}{2}\left(L + \frac{H_4}{\lambda_n^2 + H_4^2}\right) + \frac{H_3}{2}\right]\left[\frac{M}{2} + \frac{\sin(2\mu_m M)}{4\mu_m}\right]}$$

Where λ_n *and* μ_m *are positive roots of*

$$\left(H_3 H_4 - \lambda^2\right)\sin(\lambda L) + \left(H_3 + H_4\right)\lambda\cos(\lambda L) = 0$$

$$\mu\sin(\mu M) - H_1\cos(\mu M) = 0$$

<u>Solution of BVP for Poisson's Equation</u> (superposition principle):

$$u(x, y) = u_5(x, y) + u_6(x, y)$$