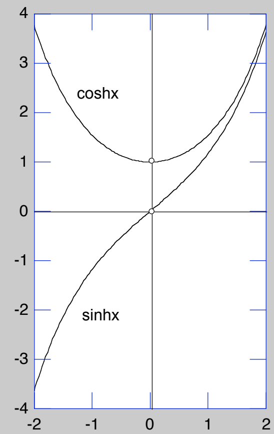
hyperbolic functions

definition	$\sinh x = \frac{e^x - e^{-x}}{2}$	$\cosh x = \frac{e^x + e^{-x}}{2}$
derivative	$\sinh' x = \cosh x$	$\cosh' x = \sinh x$
integration	$\int \sinh x\,dx = \cosh x$	$\int \cosh x\,dx = \sinh x$
symmetry	$\sinh(-x) = -\sinh x$	$\cosh(-x) = \cosh x$
value at 0	$\sinh 0 = 0$	$\cosh 0 = 1$
series expansion	$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots$	$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots$
identities	$\cosh^2 x - \sinh^2 x = 1$	$\cosh x \pm \sinh x = e^{\pm x}$



special equation

$$X''(x) - \lambda X(x) = 0$$

$\lambda =$ *constant*

auxilary equation $m^2 = \lambda$

roots $m = \pm\sqrt{\lambda}$

general solution	derivative of general solution
$X(x) = \begin{cases} c_1 e^{\sqrt{\lambda}x} + c_2 e^{-\sqrt{\lambda}x} & \lambda > 0 \\ c_1 + c_2 x & \lambda = 0 \\ c_1 \cos\sqrt{-\lambda}x + c_2 \sin\sqrt{-\lambda}x & \lambda < 0 \end{cases}$	$X'(x) = \begin{cases} c_1\sqrt{\lambda}e^{\sqrt{\lambda}x} - c_2\sqrt{\lambda}e^{-\sqrt{\lambda}x} & \lambda > 0 \\ c_2 & \lambda = 0 \\ -c_1\sqrt{-\lambda}\sin\sqrt{-\lambda}x + c_2\sqrt{-\lambda}\cos\sqrt{-\lambda}x & \lambda < 0 \end{cases}$
$X(x) = \begin{cases} c_1 \cosh\sqrt{\lambda}x + c_2 \sinh\sqrt{\lambda}x & \lambda > 0 \\ c_1 + c_2 x & \lambda = 0 \\ c_1 \cos\sqrt{-\lambda}x + c_2 \sin\sqrt{-\lambda}x & \lambda < 0 \end{cases}$	$X'(x) = \begin{cases} c_1\sqrt{\lambda}\sinh\sqrt{\lambda}x + c_2\sqrt{\lambda}\cosh\sqrt{\lambda}x & \lambda > 0 \\ c_2 & \lambda = 0 \\ -c_1\sqrt{-\lambda}\sin\sqrt{-\lambda}x + c_2\sqrt{-\lambda}\cos\sqrt{-\lambda}x & \lambda < 0 \end{cases}$
$X(x) = \begin{cases} c_1 \cosh\sqrt{\lambda}(x-x_0) + c_2 \sinh\sqrt{\lambda}(x-x_0) & \lambda > 0 \\ c_1 + c_2(x-x_0) & \lambda = 0 \\ c_1 \cos\sqrt{-\lambda}(x-x_0) + c_2 \sin\sqrt{-\lambda}(x-x_0) & \lambda < 0 \end{cases}$	$X'(x) = \begin{cases} c_1\sqrt{\lambda}\sinh\sqrt{\lambda}(x-x_0) + c_2\sqrt{\lambda}\cosh\sqrt{\lambda}(x-x_0) & \lambda > 0 \\ c_2 & \lambda = 0 \\ -c_1\sqrt{-\lambda}\sin\sqrt{-\lambda}(x-x_0) + c_2\sqrt{-\lambda}\cos\sqrt{-\lambda}(x-x_0) & \lambda < 0 \end{cases}$