laplace transform

Laplace transform	$L\{f(t)\} = F(s) = \int_0^{\infty} f(t)e^{-st}dt$	inverse Laplace transform	$f(t) = L^{-1}\{F(s)\}$

$f(t)$ is of exponential order if $|f(t)| \leq Me^{at}$ $\forall t \geq 0$ for some $a, M > 0$

existance of Laplace transform

if $f(t)$ is picewise continuos on $[0, \infty)$ and of exponential order with a and M then $L\{f(t)\}$ exists for all $s > a$

$$|L\{f(t)\}| \leq \frac{M}{s-a}$$

$L\{f(t)\} \to 0$ when $s \to \infty$

$sL\{f(t)\}$ is bounded when $s \to \infty$

PROPERTIES

linearity

$$L\{af(t) + bg(t)\} = aL\{f(t)\} + bL\{g(t)\}$$

$$L^{-1}\{\alpha F(s) + \beta G(s)\} = \alpha L^{-1}\{F(s)\} + \beta L^{-1}\{G(s)\}$$

derivative

$$L\{f'(t)\} = sF(s) - f(0)$$

$$L\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$$

$$L\{f^{(n)}(t)\} = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

integral

$$L\left\{\int_0^t f(x)dx\right\} = \frac{1}{s}F(s)$$

s-multiplied transform

$$L^{-1}\{sF(s)\} = f'(t)$$

s-divided transform

$$L^{-1}\left\{\frac{1}{s}F(s)\right\} = \int_0^t f(x)dx$$

transform differentiation

$$L\{tf(t)\} = -\frac{d}{ds}F(s)$$

$$L\{t^nf(t)\} = (-1)^n \frac{d^n}{ds^n}F(s)$$

$$L^{-1}\{F(s)\} = -\frac{1}{t}L^{-1}\{F'(s)\}$$

transform integration

$$L\left\{\frac{1}{t}f(t)\right\} = \int_s^{\infty} F(s)ds$$

$$L^{-1}\{F(s)\} = tL^{-1}\left\{\int_s^{\infty} F(s)ds\right\}$$

similarity

$$L\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right) \qquad a > 0$$

shifting on s

$$L\{e^{at}f(t)\} = F(s-a)$$

$$L^{-1}\{F(s)\} = e^{-at}L^{-1}\{F(s-a)\}$$

$$L\{e^{-at}\cos bt\} = \frac{s+a}{(s+a)^2 + b^2}$$

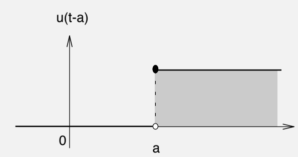
$$L\{e^{-at}\sin bt\} = \frac{b}{(s+a)^2 + b^2}$$

$$L\{e^{-at}t^n\} = \begin{cases} \dfrac{\Gamma(n+1)}{(s+a)^{n+1}} & n > -1 \\ \dfrac{n!}{(s+a)^{n+1}} & n = 0,1,\ldots \end{cases}$$

shifting on t

unit step function

$$u(t-a) = \begin{cases} 0 & t < a \\ 1 & t \geq a \end{cases}$$



$$L\{u(t-a)f(t)\} = e^{-as}F(s)$$

$$L^{-1}\{e^{-as}F(s)\} = u(t-a)f(t-a) \qquad a \geq 0$$

$$L\{u(t-a)f(t)\} = e^{-as}L\{f(t+a)\}$$

convolution

$$f * g = \int_0^t f(t-x)g(x)dx$$

transform of convolution

$$L\{f * g\} = L\{f\}L\{g\}$$