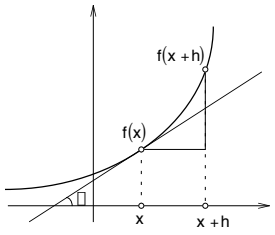
differentiation

derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

if limit exists then
function f(x) is differentiable and
f'(x) is a derivative of function f(x)



equation of tangent line $y = f'(x_0)(x - x_0) + f(x_0)$

$f'(x) = m$ slope of tangent line

$= \tan\theta$

differentiation rules

$(cu)' = cu'$

$(u + v)' = u' + v'$

$(u - v)' = u' - v'$

$(uv)' = uv' + vu'$

$(uvw)' = uvw' + uvw' + uvw'$

$\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$

$\left(\frac{1}{v}\right)' = \frac{-v'}{v^2}$

chain rule

$y = f[u(x)]$

$$\frac{dy}{dx} = \frac{df}{du}\frac{du}{dx}$$

example: $y = \sin(x^2)$ $u = x^2$

$$y' = \frac{d(\sin u)}{du}\frac{du}{dx} = (\cos u)(2x) = 2x\cos(x^2)$$

derivative of inverse function

$y = f(x)$

$y = f^{-1}(x)$

$$\left[f^{-1}(x)\right]' = \frac{1}{f'\left[f^{-1}(x)\right]}$$

1) find $f'(x)$

2) write $\frac{1}{f'(x)}$

3) replace x by $f^{-1}(x)$ and simplify expression

example:

$f(x) = e^x$

$f^{-1}(x) = \ln x$

1) find $f'(x) = e^x$

2) write $\frac{1}{f'(x)} = \frac{1}{e^x}$

3) replace x by and simplify expression

$$(\ln x)' = \left[f^{-1}(x)\right]' = \frac{1}{f'\left[f^{-1}(x)\right]} = \frac{1}{e^{\ln x}} = \frac{1}{x}$$

implicit differentiation

given implicit function $f(x,y) = 0$

find y'

differentiate using differentiation rules

terms with x only differetiate as function of x

terms with y differentiate with the chain rule $\frac{d}{dx}g(y) = \frac{dg}{dy}y'$

solve for y'

example: $f(x,y) = xy + \sin y - 1 = 0$

$(xy)' + (\sin y)' - (1)' = 0$

$y + xy' + (\cos y)y' = 0$

$y' = \frac{-y}{x + \cos y}$

table of derivatives

$f(x)$	$f'(x)$	$f(x)$	$f'(x)$	$f(x)$	$f'(x)$
c	0	$\sin x$	$\cos x$	$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
x	1	$\cos x$	$-\sin x$	$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$
x^n	nx^{n-1}	$\tan x$	$1 + \tan^2 x$		
			$\frac{1}{\cos^2 x}$	$\tan^{-1} x$	$\frac{1}{1+x^2}$
e^x	e^x		$\sec^2 x$		
		$\cot x$	$-1 - \cot^2 x$	$\cot^{-1} x$	$\frac{-1}{1+x^2}$
a^x	$a^x \ln a$		$\frac{-1}{\sin^2 x}$		
			$-\csc^2 x$	$\csc^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$
$\ln x$	$\frac{1}{x}$	$\csc x$	$-\csc x \cot x$		
				$\sec^{-1} x$	$\frac{1}{x\sqrt{x^2-1}}$
$\log_a x$	$\frac{1}{x \ln a}$	$\sec x$	$\sec x \tan x$		