

1.3 Classification of Differential Equations

Differential equation

ODE

PDE

Systems of differential equations

Order of ODE

Normal ODE

Linear ODE

Non-Linear ODE

Solution of ODE

IVP

Existence of a solution

General solution

Particular solution

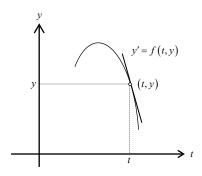
Uniqueness of a solution

1.1 Direction fields

Consider the differential equation of the first order explicitly written for the derivative of unknown function y'

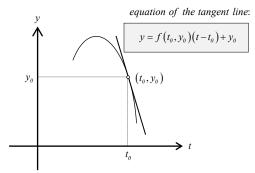
$$\frac{dy}{dt} = f\left(t, y\right)$$

where f(t, y) is some given function of variables t and y. Then we say that equation is written in **normal form**. Derivative y'(t) defines the slope of the tangent line to the curve y = y(t) at the point t.



If derivative y'(t) is given by the differential equation in normal form y' = f(t, y),

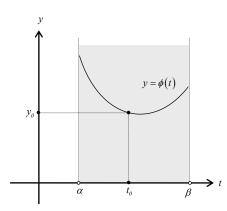
then equation the tangent line to the curve y = y(t) at some fixed point (t_0, y_0) can be written as



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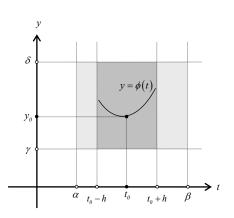
2.4 Differences between linear and non-linear equations

Theorem 2.4.1 (existence and uniqueness of the solution of IVP for linear 1st order ODE y' + p(t)y = g(t))



Let $t_0 \in (\alpha, \beta)$ and let $p(t), g(t) \in C(\alpha, \beta)$ (continuous functions) then the linear differential equation y' + p(t)y = g(t)has a unique solution $y = \phi(t), t \in (\alpha, \beta)$ such that $y(t_0) = y_0$

Theorem 2.4.2 (existence and uniqueness of the solution of IVP for **non-linear** 1st order ODE y' = f(y,t))



Let $f(y,t), \frac{\partial f(y,t)}{\partial y} \in C((\alpha,\beta) \times (\gamma,\delta))$ (continuous)

and let $t_0 \in (\alpha, \beta)$ and $y_0 \in (\gamma, \delta)$

then the non-linear differential equation y' = f(y,t)has a unique solution $y = \phi(t)$, $t \in (t_0 - h, t_0 + h) \subseteq (\alpha, \beta)$, h > 0such that $y(t_0) = y_0$. Here, h > 0 is some positive number.

Remark: if only $f(y,t) \in C((\alpha,\beta) \times (\gamma,\delta))$ is continuous, then solution of IVP exists but is not necessarily unique.

the theorems guarantee only that under given conditions there exists a unique solution of the IVP, but they do not claim that the solution does not exist if the conditions of the theorems are violated.

| $y' = f\left(x, y\right)$ | if $f(x,b) = 0$, then $y = b$ is a solution |
|---------------------------|---|
| x' = g(x, y) | if $g(a, y) = 0$, then $x = a$ is a solution |
| F(x, y, c) = 0 | Solution which includes an arbitrary constant. |
| | General solution of linear ODE includes all possible solutions (complete solution). |
| | For non-linear ODE, there can be some |
| | additional solutions. |
| | |

Note:

Constant solutions

General Solution (implicit)

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Suppressed solutions
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Solutions not described by the general solution



2.1 Linear ODE – Integrating Factor

| Standard form | y' + p(t)y = g(t) | Initial Condition: $y(t_0) = y_0$ |
|--------------------------------|---|--|
| Integrating factor | $\mu(t) = e^{\int p(t)dt}$ | |
| General solution | $y = \frac{c}{\mu(t)} + \frac{l}{\mu(t)} \int \mu(t) g(t) dt$ | |
| | $y = \frac{c}{\mu(t)} + \frac{l}{\mu(t)} \int_{t_0}^{t} \mu(s) g(s) ds$ | (integral form of solution) |
| Solution of IVP | $y = y_0 \frac{\mu(t_0)}{\mu(t)} + \frac{1}{\mu(t)} \int_{t_0}^t \mu(s) g(s)$ | ds |
| Case of a constant coefficient | y' + ay = g(t) | |
| Integrating factor | $\mu(t) = e^{at}$ | |
| General solution | $y = ce^{-at} + e^{-at} \int e^{at} g(t) dt$ | |
| Solution of IVP | $y = y_0 e^{-a(t-t_0)} + e^{-at} \int_{t_0}^t e^{as} g(s) ds$ | |
| Case $g(t) = b = const$ | y' + ay = b | у |
| General solution | $y = ce^{-at} + \frac{b}{a}$ | b/a graph in |

Solution of IVP

Exercise:

Solve

 $ty' + 2y = 4t^2$ y(1) = 2

 $y = e^{-a(t-t_0)} \left(y_0 - \frac{b}{a} \right) + \frac{b}{a}$

and sketch the solution curve

 t_0

 (t_0, y_0)

 y_0

a case of a,b > 0

→ t

Example:

1st Order ODE

Find a general solution of equation

 $y' + (\cot x)y = \sin 2x$

and sketch the solution curves.

Solution:

The integrating factor for this equation is

$$\mu(x) = e^{\int \cot x \, dx} = e^{\ln|\sin x|} = \sin x$$

Then a general solution is

$$y = \frac{c}{\sin x} + \frac{1}{\sin x} \int \sin(x) \sin(2x) dx$$

= $\frac{c}{\sin x} + \frac{2}{\sin x} \int \sin(x) \sin(x) \cos(x) dx$ (double angle formula)
= $\frac{c}{\sin x} + \frac{2}{\sin x} \int \sin^2(x) d\sin(x)$ (u-substitution)
= $\frac{c}{\sin x} + \frac{2 \sin^2 x}{3}$

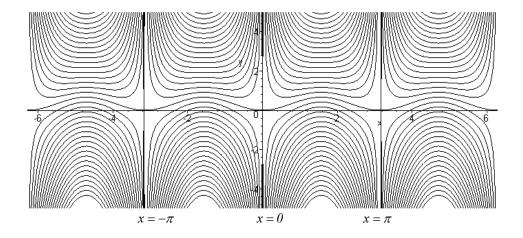
Maple:

create a sequence of particular solutions by varying the constant c, and then plot the graph of solution curves:

>
$$y(x) := 2 \times \sin(x)^2/3 + c/\sin(x);$$

 $y(x) := \frac{2}{3} \sin(x)^2 + \frac{c}{\sin(x)}$

 $> p:=\{seq(subs(c=i/4,y(x)), i=-20..20)\}:$



| Math-303 | Chapters 1-2 | 1 st Order ODE | September 20, 2019 8 |
|----------|--|---|---|
| 2.2 | Separable equation | | |
|] | Differential form of ODE | M(x,y)dx + N(x,y)dy = 0 | Note that equation in differential form has no distinction between independent and dependent variable |
| | differential form is equivalent to a pair of differential equations | $M(x, y)\frac{dx}{dy} + N(x, y) = 0$ $M(x, y) + N(x, y)\frac{dy}{dx} = 0$ | |
| 5 | Separable ODE | M(x)dx + N(y)dy = 0 | |
| | General Solution | $\int M(x)dx + \int N(y)dy = c$ | |
| 5 | Solution of IVP $y(x_0) = y_0$ | $\int_{y_0}^{x} M(x) dx + \int_{y_0}^{y} N(y) dy = 0$ | |

| Equation is <i>homogeneous</i> of order <i>m</i> if | | $) = \lambda^{m} M(x, y)$ $= \lambda^{m} N(x, y)$ | are homogeneous functions of order <i>m</i> |
|--|----------------------|---|---|
| Homogeneous equation can be reduc | ed to <i>separab</i> | le | Back substitution: |
| by a change of variable y to | y = ux | dy = udx + xdu | $u = \frac{y}{x}$ |
| or by a change of variable x to | x = vu | dx = vdy + ydv | $v = \frac{x}{y}$ |

By change to polar coordinates $x = r \cos \theta$ $y = r \sin \theta$

| Example | Problem 2.2 #1, p.47: | Solve $y' = x^2/y$ | subject to initial condition | y(3) = 2 |
|---------|-----------------------|--------------------|------------------------------|----------|
| Example | Problem 2.2 #31 | | | |

Math-303 Chapters 1-2 1st Order ODE September 20, 2019 9 Solve the differential equation $y^2 + 2x^2 + xyy' = 0$ Example $(y^2 + 2x^2)dx + xydy = 0$ differential form Solution: M and N are homogeneous functions of degree 2. Change of variable: dy = xdu + udxy = ux $(u^{2}x^{2} + 2x^{2})dx + xux(xdu + udx) = 0$ $\left(u^{2}x^{2} + 2x^{2} + u^{2}x^{2}\right)dx + ux^{3}du = 0$ $2x^{2}(u^{2}+1)dx + ux^{3}du = 0$ separable $2\frac{dx}{x} + \frac{udu}{u^2 + 1} = 0$ separate variables, $x \neq 0$ $2\frac{dx}{x} + \frac{1}{2}\frac{d(u^2 + 1)}{u^2 + 1} = 0$ integrate $ln x^{4} + ln(u^{2} + 1) = ln c$ solution $x^4 \left(u^2 + l \right) = c$ back substitution $u = \frac{y}{x}$ $\left(y^2 + x^2\right)x^2 = c$ general solution(implicit)

Check for suppressed solutions: earlier we assumed that $x \neq 0$, then check that

but this solution is a particular case of general solution when c = 0.

x = 0

Use Maple to plot the solution curves:

> f:={seq(x^2*(y^2+x^2)=i/8,i=0..12)}: > implicitplot(f,x=-2..2,y=-5..5)

2.6 Exact Equations and Integrating Factors

| Differential of $f(x, y)$ is | $d f(x,y) = \frac{\partial f(x,y)}{\partial x} dx + \frac{\partial f(x,y)}{\partial y} dy$ |
|-------------------------------------|---|
| Exact equation | M(x, y)dx + N(x, y)dy = 0 is <i>exact</i> if there exists some $f(x, y)$ such that |
| | d f(x, y) = M(x, y) dx + N(x, y) dy |
| Test on exact equation | $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ |
| General solution | f(x, y) = c level curves of the surface defined by $f(x, y)$ |
| | |
| Finding $f(x, y)$ | 1) $\frac{\partial f}{\partial x} = M(x, y) \implies f(x, y) = \int M(x, y) dx + k(y)$ |
| | 2) $\frac{\partial f}{\partial y} = N(x, y) \implies \frac{\partial}{\partial y} \int M(x, y) dx + \frac{d}{dy} k(y) = N(x, y)$ |
| | \Rightarrow find $k(y)$ |
| | 3) General Solution: $f(x,y) = \int M(x,y) dx + k(y) = c$ |

Integrating factor **µ**

Equation multiplied by an integration factor μ becomes **exact**.

i) if
$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = h(x)$$
 then $\mu(x) = e^{\int h(x)dx}$
ii) if $\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = g(y)$ then $\mu(y) = e^{\int g(y)dy}$

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Example (p.95)

Solve

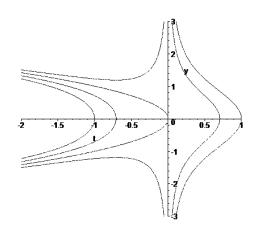
 $2x + y^2 + 2xyy' = 0$

Rewrite in differential form
$$(2x + y^2) dx + 2xy dy = 0$$

Test for exact: $\frac{\partial M}{\partial y} = \frac{\partial (2x + y^2)}{\partial y} = 2y$
 $\frac{\partial N}{\partial x} = \frac{\partial (2xy)}{\partial x} = 2y \implies \text{Exact}$

Find
$$f(x, y)$$

1) $\frac{\partial f}{\partial x} = M$
 $\frac{\partial f}{\partial x} = 2x + y^2$
 $f = x^2 + y^2 x + k(y)$
2) $\frac{\partial f}{\partial y} = N$
 $\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} \left[x^2 + y^2 x + k(y) \right] = 2yx + \frac{dk}{dy}$
 $2yx + \frac{dk}{dy} = 2xy$
 $\frac{dk}{dy} = 0$
 $k = c$
Therefore, $f = x^2 + y^2 x + c$
 $x^2 + y^2 x + c = c_j$ combine constants, then
General solution: $(x + y^2)x = c$



Math. 203 (hptop 1.2)
 Part ODE
 Segmet 2.9.201 (1)

 Second derivative

$$y' = f(y)$$
 independent variable is that is not a equation explicitly.

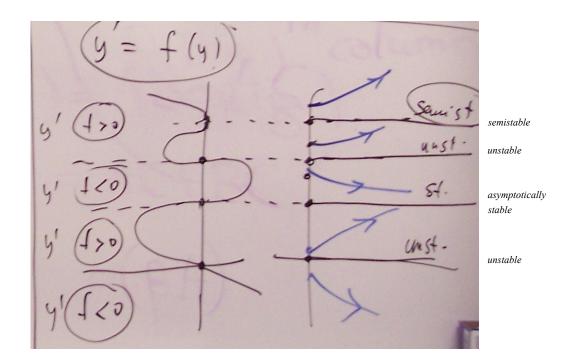
 Second derivative
 $y' = f(y)$
 eritical points $y - y_x$ are constant solutions. They are called the equilibrium solutions.

 Second derivative
 $y' = \frac{\partial}{\partial t} y' = \frac{\partial}{\partial t} f[y(t)]^{-\frac{d^2}{200}} \frac{df}{dy'} \frac{df}{dy'}$

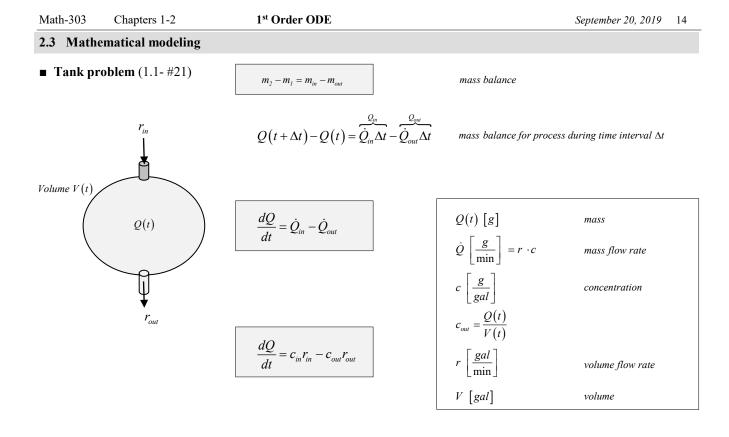
Second derivative
$$y'' = f \cdot \frac{df}{dy}$$
 $\frac{df}{dy} = r\left(1 - \frac{2}{K}y\right)$
 $y'' = r^2 y \left(1 - \frac{1}{K}y\right) \left(1 - \frac{2}{K}y\right)$ $y = 0, y = K$ and $y = \frac{K}{2}$

Stability of Autonomous Equation

 $y = y_k$, $\{y_k: f(y_k) = 0\}$ stability of equilibrium solutions



| <i>Example:2.5 #22</i> | $\frac{dy}{dt} = \alpha y (1 - y)$ | $y(0) = y_0$ | $\alpha > 0$ |
|------------------------|------------------------------------|--------------|--------------|
|------------------------|------------------------------------|--------------|--------------|

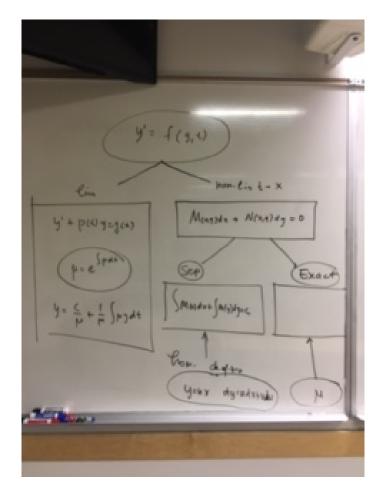


for $r_{in} = r_{out} = r = const$ and V = const

$$\frac{dQ}{dt} = -\frac{r}{V}Q + c_{in}r$$

| Exponential decay (1.2-12,13) | $\frac{dQ}{dt} = -rQ$ | r > 0 | |
|---|------------------------------------|--------------|---|
| Exponential growth | $\frac{dQ}{dt} = rQ$ | r > 0 | |
| • Population model (1.1, p.5) | $\frac{dp}{dt} = rp - k$ | r, k > 0 | 1 |
| Newton's Law of Cooling (1.1-23, 1.2-15) | $\frac{du}{dt} = -k \cdot (u - T)$ | <i>k</i> > 0 | |

| • Exponential decay $\frac{dp}{dt} = -rp$ $r \left[\frac{1}{sw}\right] = decay rate, r > 0$ • Exponential growth $\frac{dp}{dt} = rp$ $r \left[\frac{1}{sw}\right] = growth rate, r > 0$ • Tank problem $Q(r)(flow)$ awavas of salt $\frac{dQ}{dt} = -\frac{r}{V}Q + c_n r$ • Population model (mice-owl) $p(r)[m(r])$ powers rate $r \left[\frac{m}{swr}\right] = growth rate, r > 0$ • Reputation model (mice-owl) $p(r)[m(r])$ powers rate $r \left[\frac{m}{swr}\right] = growth rate, r > 0$ • Bank model S(i) [S] = investment of defa $r \left[\frac{f_{swr}}{f_{swr}}\right] = continuous annual k = 12 \cdot m$ $\frac{dS}{dt} = rS + k$ $S(t) = S_0 e^n + \frac{k}{r}(e^n - 1)$ $w \left[\frac{S}{swr}\right] = continuous annual k = 12 \cdot m$ $\frac{dS}{dt} = rS - w$ $S(t) = S_0 e^n - \frac{12m}{r}(e^n - 1)$ $w \left[\frac{S}{swr}\right] = continuous annual k = 12 \cdot m$ $\frac{dS}{dt} = rS - w$ $S(t) = S_0 e^n - \frac{12m}{r}(e^n - 1)$ $w \left[\frac{S}{swr}\right] = continuous annual k = 12 \cdot m$ $\frac{dS}{dt} = rS - p$ $S(t) = S_0 e^n - \frac{12m}{r}(e^n - 1)$ $w \left[\frac{S}{swr}\right] = continuous annual k = 12 \cdot m$ $\frac{dS}{dt} = rS - p$ $S(t) = S_0 e^n - \frac{12m}{r}(e^n - 1)$ $w \left[\frac{S}{swr}\right] = rate of proments w = 12 \cdot m \frac{dS}{dt} = rS - p S(t) = S_0 e^n - \frac{12m}{r}(e^n - $ | $r\left[\frac{l}{s\infty}\right] = decay rate, r > 0$ a Exponential growth $\frac{dp}{dt} = rp$ $r\left[\frac{l}{s\infty}\right] = growth rate, r > 0$ b Tank problem $Q(r)[tbm] \text{ somulo of sult} \qquad \frac{dQ}{dt} = -\frac{r}{V}Q + c_{n}r$ c Population model (mice-owl) $p(r)[fixe] \text{ superiments or definition } \qquad \frac{dp}{dt} = rp - k$ $r\left[\frac{l}{swr}\right] = growth rate rate reproduction rate of deposition reproduction rate rate of reproduction rate reproduction rate rate of deposition reproduction rate reproduction rate rate of deposition rate reproduction rate rate of withdraws we reproduct rate of \frac{dS}{dt} = rS - w S(t) = S_0e^{ret} - \frac{W}{r}(e^{rt} - 1) ret = \int_{month} deposition rate rate of payments S(t) = [S_0 - \frac{12m}{r}(e^{rt} - 1) - \frac{dS}{dt} = rS - p S(t)$ | | | | |
|---|---|---|--|---|---|
| $r\left[\frac{1}{yw}\right] = decay rate, r > 0$ Exponential growth $\frac{dp}{dt} = rp$ $r\left[\frac{1}{yw}\right] = growth rate, r > 0$ Finally problem $Q(t)[bbs] amount of salt$ $\frac{dQ}{dt} = -\frac{r}{r}Q + c_{n}r$ Population model (mice-owl) $p(t)[mix] population$ $\frac{dp}{dt} = rp - k$ $r\left[\frac{1}{ywr}\right] = growth rate$ $\frac{dp}{dt} = rp - k$ $r\left[\frac{1}{ywr}\right] = growth rate$ $\frac{dp}{dt} = rp - k$ $r\left[\frac{1}{ywr}\right] = growth rate$ $\frac{dp}{dt} = rp - k$ $r\left[\frac{1}{ywr}\right] = growth rate$ $\frac{dp}{dt} = rp - k$ $r\left[\frac{1}{ywr}\right] = growth rate$ $\frac{dp}{dt} = rp - k$ $r\left[\frac{1}{ywr}\right] = growth rate$ $\frac{dp}{dt} = rp - k$ $r\left[\frac{1}{ywr}\right] = growth rate$ $\frac{dp}{dt} = rp - k$ $r\left[\frac{1}{ywr}\right] = growth rate$ $rate of vibility mice$ $p(0) = p, [mix2] means and intercer rate$ $rate of diations and intercer rate$ $rate of diations and intercer rate$ $rate of diations is k = 12 \cdot m$ $\frac{dS}{dt} = rS - w$ $S(t) = S_{0}e^{rt} - \frac{h^{2}}{r}(e^{rt} - 1)$ $P\left[\frac{s}{ywr}\right] = continuous and its w = 12 \cdot m$ $\frac{dS}{dt} = rS - p$ $S(t) = S_{0}e^{rt} - \frac{p}{r}(e^{rt} - 1)$ $S(t) = S_{0}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $P\left[\frac{s}{ywr}\right] = continuous and w = 12 \cdot m$ $\frac{dS}{dt} = rS - p$ $S(t) = S_{0}e^{rt} - \frac{p}{r}(e^{rt} - 1)$ $S(t) = S_{0}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $m\left[\frac{s}{ywr}\right] = continuous and w = 12 \cdot m$ $\frac{dS}{dt} = rS - p$ $S(t) = S_{0}e^{rt} - \frac{p}{r}(e^{rt} - 1)$ $S(t) = S_{0}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $S(t) = \left(S_{0} - \frac{12m}{r}(e^{rt} - 1)\right)$ $S(t) = \left(S_{0} - \frac{12m}{r}(e^{rt}$ | $r\left[\frac{1}{rec}\right] - decay rate, r > 0$ $r\left[\frac{1}{rec}\right] - decay rate, r > 0$ $\frac{du}{dt} = rp$ $r\left[\frac{1}{rec}\right] = growth rate, r > 0$ $\frac{du}{dt} = rp$ $r\left[\frac{1}{rec}\right] = growth rate, r > 0$ $\frac{du}{dt} = rp$ $\frac{du}{dt} = rp$ $\frac{du}{dt} = rp - k$ $r\left[\frac{1}{rec}\right] = growth rate, r > 0$ $\frac{du}{dt} = rp - k$ $r\left[\frac{1}{rec}\right] = growth rate, r > 0$ $\frac{du}{dt} = rp - k$ $r\left[\frac{1}{rec}\right] = growth rate, r > 0$ $\frac{du}{dt} = rp - k$ $r\left[\frac{1}{rec}\right] = growth rate, r > 0$ $\frac{du}{dt} = rp - k$ $r\left[\frac{1}{rec}\right] = growth rate, r > 0$ $\frac{du}{dt} = rp - k$ $r\left[\frac{1}{rec}\right] = growth rate, r > 0$ $\frac{du}{dt} = rp - k$ $r\left[\frac{1}{rec}\right] = growth rate, r > 0$ $\frac{du}{dt} = rp - k$ $r\left[\frac{1}{rec}\right] = growth rate, r > 0$ $\frac{du}{dt} = rp - k$ $r\left[\frac{1}{rec}\right] = growth rate, r > 0$ $\frac{du}{dt} = rp - k$ $r\left[\frac{1}{rec}\right] = growth rate, r > 0$ $\frac{du}{dt} = rp - k$ $r\left[\frac{1}{rec}\right] = growth rate, r > 0$ $\frac{du}{dt} = rp - k$ $r\left[\frac{1}{rec}\right] = growth rate, r > 0$ $\frac{du}{dt} = rp - k$ $r\left[\frac{1}{rec}\right] = growth rate, r > 0$ $\frac{du}{dt} = rp - k$ $r\left[\frac{1}{rec}\right] = growth rate, r > 0$ $\frac{du}{dt} = rp - k$ $r\left[\frac{1}{rec}\right] = growth rate, r > 0$ $\frac{du}{dt} = rp - k$ $r\left[\frac{1}{rec}\right] = growth rate, r > 0$ $\frac{du}{dt} = rp - k$ $r\left[\frac{1}{rec}\right] = growth rate, r > 0$ $\frac{du}{dt} = rs + k$ $S(t) = S_{0}e^{t} + \frac{k}{r}(e^{t} - t)$ $S(t) = S_{0}e^{t} + \frac{12m}{r}(e^{t} - t)$ $\frac{1}{r}\left[\frac{1}{rec}\right] = growth rate, r > 0$ $\frac{du}{dt} = rS - p$ $S(t) = S_{0}e^{t} - \frac{12m}{r}(e^{t} - t)$ $\frac{du}{dt} = rS - p$ $S(t) = S_{0}e^{t} - \frac{12m}{r}(e^{t} - t)$ $\frac{du}{dt} = rS - p$ $S(t) = S_{0}e^{t} - \frac{12m}{r}(e^{t} - t)$ $\frac{du}{dt} = rS - p$ $S(t) = S_{0}e^{t} - \frac{12m}{r}(e^{t} - t)$ $\frac{du}{dt} = rS - p$ $S(t) = S_{0}e^{t} - \frac{12m}{r}(e^{t} - t)$ $\frac{du}{dt} = rk \cdot (a - T)$ $\frac{du}{dt} = -k \cdot (a - T)$ $\frac{du}{dt} = -\frac{r}{m} + \frac{r}{m}$ $\frac{du}{dt} = -\frac{r}{m} + \frac{r}{m}$ | Exponential decay | $\frac{dp}{dt} = -rp$ | | |
| • Even • Exponential growth $\frac{dp}{dt} = rp$ $r\left[\frac{1}{rec}\right] = growth rate, r > 0$ • Tank problem $Q(t)[thm] anaant of salt \frac{dQ}{dt} = -\frac{r}{V}Q + c_n r• Population model (mice-owl)p(t)[mice] population \frac{dp}{dt} = rp - kr\left[\frac{1}{rec}\right] = growth rate r\left[\frac{1}{rec}\right] = growth rate rate of teams r\left[\frac{1}{rec}\right] = growth rate rate of teams r\left[\frac{1}{rec}\right] = growth rate rate of teams r\left[\frac{1}{rec}\right] = rrccrate of teams r\left[\frac{1}{rec}\right] = rrcccrate of teams rate of teamsrate of teams rate of teams rate of teams rate of teamsrate of teams rate of teams rate of teams rate of teamsrate of teams rate of teamsrate of teams rate of teams rate of teamsrate of teamsrate of teamsrate of teams rate of teamsrate of tea$ | • Exponential growth $\frac{dp}{dt} = rp$ • $\left[\frac{t}{ sw }\right] = growth rate, r > 0$ • Tank problem $\mathcal{Q}(t)(bion)$ amount of sub $\frac{dQ}{dt} = -\frac{r}{r}Q + c_{w}r$ • Population model (mice-owl) $p(t)(mere)$ providering $\frac{dp}{dt} = rp - k$ $r\left[\frac{t}{ sw }\right] = reproduction \frac{dp}{dt} = rp - kr\left[\frac{t}{ sw }\right] = reproduction rate k \left[\frac{mi}{ sw }\right] = reproduction rate k \left[\frac{mi}{ sw }\right] = restinguestion rate k \left[\frac{mi}{ sw }\right] = restinguestion rate k \left[\frac{t}{ sw }\right] = restinguestion rate p = 12 \cdot m \frac{dS}{dt} = rS - w S(t) = S_{0}e^{t} - \frac{t}{r}(e^{t} - t) S(t) = S_{0}e^{t} - \frac{t2m}{r}(e^{t} - t)m \left[\frac{s}{ sw }\right] = restinguestion rate p = 12 \cdot m \frac{dS}{dt} = rS - p S(t) = S_{0}e^{t} - \frac{t2m}{r}(e^{t} - t)m \left[\frac{s}{ sw }\right] = -restingerstingsS(t) = \left[\frac{s}{s}e^{t} - \frac{t2m}{r}(e^{t} - t)\right]S(t) = \left[\frac{s}{s}e^{t} - \frac{t2m}{r}(e^{t} - t)\right]m \left[\frac{s}{ sw }\right] = -restingerstingsS(t) = \left[\frac{s}{s}e^{t} - \frac{t2m}{r}(e^{t} - t)\right]S(t) = \left[\frac{s}{s}e^{t} - \frac{t2m}{r}(e^{t} - t)\right]F_{t} = mgm \left[\frac{t}{t}e^{t}e^{t}e^{t}e^{t} + \frac{t2m}{r}\right]F_{t} = mg\frac{dt}{dt} = -k \cdot (m - T)k \left[\frac{t}{ s }e^{t}e^{t}e^{t}e^{t}e^{t}e^{t}e^{t}e^{t$ | $r\left[\frac{l}{l}\right] = decay rate, r > 0$ | ai | | |
| • Tank problem $Q(i)[tm] \text{ amount of valt} \qquad \qquad$ | • Tank problem $Q(t)[tbm] \text{ amons of salt} \qquad \frac{dQ}{dt} = -\frac{r}{r}Q + c_m r$ • Population model (mice-oxit) $p(t)[mice] population p(t)[mice] population response to the equation and \frac{dp}{dt} = rp - k r\left[\frac{1}{ywr}\right] = \frac{growh rate}{reproduction rate} k\left[\frac{mice}{1ywr}\right] = \frac{growh rate}{rete of population} • Bank modelS(t) [S] = investment or defitr\left[\frac{1}{ywr}\right] = \frac{growh rate}{rate of return} k = 12 \cdot m \frac{dS}{dt} = rS + k \qquad S(t) = S_0e^n + \frac{k}{r}(e^n - 1) \qquad S(t) = S_0e^n + \frac{12m}{r}(e^n - 1) w\left[\frac{S}{1ywr}\right] = \frac{growh rate}{rate of withdrawis} \qquad w = 12 \cdot m \qquad \frac{dS}{dt} = rS - w \qquad S(t) = S_0e^n - \frac{w}{r}(e^n - 1) \qquad S(t) = S_0e^n - \frac{12m}{r}(e^n - 1) p\left[\frac{S}{1ywr}\right] = \frac{growh rate}{rate of withdrawis} \qquad w = 12 \cdot m \qquad \frac{dS}{dt} = rS - p \qquad S(t) = S_0e^n - \frac{p}{r}(e^n - 1) \qquad S(t) = S_0e^n - \frac{12m}{r}(e^n - 1) m\left[\frac{S}{1ywr}\right] = \frac{growh rate}{rate of population} \qquad p = 12 \cdot m \qquad \frac{dS}{dt} = rS - p \qquad S(t) = S_1e^n - \frac{p}{r}(e^n - 1) \qquad S(t) = S_0e^n - \frac{12m}{r}(e^n - 1) m\left[\frac{S}{1ywr}\right] = \frac{wh whareh or population}{rate of population} \qquad S(t) = \left[\frac{S_0 - \frac{12m}{r}(e^n - 1)}{r}e^n + \frac{12m}{r}\right] S(0) - S_1[S] initial deposition or a low \qquad S(t) = \frac{12m}{r}e^n + \frac{dw}{dt} = -k \cdot (w - T) 4\left[\frac{1}{\sqrt{s}}e^{-1}\right] = \frac{muc}{r}e^{-1} = \frac{muc}{r}e^{-1} = \frac{w}{r}e^{-1} = \frac{w}{r$ | | $\frac{dp}{dt} = rp$ | | |
| $Q(t)[tm] \text{ anount of salt} \qquad \qquad \frac{dQ}{dt} = -\frac{r}{r}Q + c_n r$ Population model (mice-owl) $p(t)[mice] \text{ population} \qquad \qquad \frac{dp}{dt} = rp - k$ $r\left[\frac{1}{ywr}\right] = \frac{growh rate}{reproduction rate}$ $k\left[\frac{mice}{ywr}\right] = \frac{continuous rate}{of killing mice}$ $p(0) = p_n [mice] \text{ initial population}$ Bank model $S(t) [S] = investment or debt$ $r\left[\frac{1}{ywr}\right] = \frac{continuous annual}{rate of return}$ $k\left[\frac{S}{ywar}\right] = \frac{continuous annual}{rate of deposits} k = 12 \cdot m$ $\frac{dS}{dt} = rS + k$ $S(t) = S_0 e^n + \frac{k}{r}(e^n - 1)$ $S(t) = S_0 e^n + \frac{12m}{r}(e^n - 1)$ $w\left[\frac{S}{ywar}\right] = \frac{continuous annual}{rate of deposits} w = 12 \cdot m$ $\frac{dS}{dt} = rS - w$ $S(t) = S_0 e^n - \frac{w}{r}(e^n - 1)$ $S(t) = S_0 e^n - \frac{12m}{r}(e^n - 1)$ $p\left[\frac{S}{ywar}\right] = \frac{continuous annual}{rate of poyments} p = 12 \cdot m$ $\frac{dS}{dt} = rS - p$ $S(t) = S_0 e^n - \frac{p}{r}(e^n - 1)$ $S(t) = S_0 e^n - \frac{12m}{r}(e^n - 1)$ $m\left[\frac{S}{ywar}\right] = \frac{continuous annual}{rate of poyments} p = 12 \cdot m$ $\frac{dS}{dt} = rS - p$ $S(t) = S_0 e^n - \frac{p}{r}(e^n - 1)$ $S(t) = S_0 e^n - \frac{12m}{r}(e^n - 1)$ $m\left[\frac{S}{ywar}\right] = \frac{continuous annual}{rate of poyments} p = 12 \cdot m$ $\frac{dS}{dt} = rS - p$ $S(t) = S_0 e^n - \frac{p}{r}(e^n - 1)$ $S(t) = S_0 e^n - \frac{12m}{r}(e^n - 1)$ $m\left[\frac{S}{ywar}\right] = \frac{continuous annual}{rate of poyments} p = 12 \cdot m$ $\frac{dS}{dt} = rS - p$ $S(t) = S_0 e^n - \frac{p}{r}(e^n - 1)$ $S(t) = S_0 e^n - \frac{12m}{r}(e^n - 1)$ $m\left[\frac{S}{ywar}\right] = \frac{continuous annual}{rate of poyments} p = 12 \cdot m$ $\frac{dS}{dt} = rS - p$ $S(t) = S_0 e^n - \frac{p}{r}(e^n - 1)$ $S(t) = S_0 e^n - \frac{12m}{r}(e^n - 1)$ $S(t) = \left(S_0 - \frac{12m}{r}\right)e^n + \frac{12m}{r}$ | $\begin{array}{c} Q(t)[km] \ \text{ansame of solt} & \frac{dQ}{dt} = -\frac{r}{V}Q + c_m r \\ \end{array}$ $\begin{array}{c} \textbf{Population model (mice-owl)} \\ p(t)[mice] population & \frac{dp}{dt} = rp - k \\ r\left[\frac{1}{2wr}\right] = \frac{growh rate}{rgrowhetron rate} \\ \frac{mice}{12wr} = \frac{growh rate}{rgrowhetron rate} \\ \frac{mice}{12wr} = \frac{continuous rate}{continuous rate} \\ \frac{mice}{12wr} = \frac{continuous rate}{continuous rate} \\ r(0) - p_n [mice] initial population \\ \textbf{S}(t) [S] = investment or deht \\ r\left[\frac{1}{2wr}\right] = \frac{continuous rate}{continuous rate} \\ \frac{1}{12wr} = \frac{continuous rate}{cate of deposits} k = t2 \cdot m \frac{dS}{dt} = rS + k \\ S(t) = S_n e^{at} + \frac{k}{r}(e^{at} - 1) \\ \textbf{W} \left[\frac{S}{12wr}\right] = \frac{continuous rate}{cate of deposits} k = t2 \cdot m \frac{dS}{dt} = rS - w \\ \textbf{S}(t) = S_n e^{at} - \frac{k}{r}(e^{at} - 1) \\ \textbf{W} \left[\frac{S}{12wr}\right] = \frac{continuous namual}{rate of vibiliarvits} w = t2 \cdot m \frac{dS}{dt} = rS - w \\ S(t) = S_n e^{at} - \frac{k}{r}(e^{at} - 1) \\ p\left[\frac{S}{12wr}\right] - \frac{continuous namual}{rate of vibiliarvits} w = t2 \cdot m \frac{dS}{dt} = rS - p \\ S(t) = S_n e^{at} - \frac{p}{r}(e^{at} - 1) \\ S(t) = S_n e^{at} - \frac{12m}{r}(e^{at} - 1) \\ p\left[\frac{S}{12wr}\right] - \frac{continuous namual}{rate of vibiliarvits} w = t2 \cdot m \frac{dS}{dt} = rS - p \\ S(t) = S_n e^{at} - \frac{p}{r}(e^{at} - 1) \\ S(t) = S_n e^{at} - \frac{12m}{r}(e^{at} - 1) \\ m\left[\frac{S}{12wr}\right] - \frac{continuous namual}{rate of pupments} p - t2 \cdot m \\ \frac{dS}{dt} = rS - p \\ S(t) = S_n e^{at} - \frac{p}{r}(e^{at} - 1) \\ S(t) = S_n e^{at} - \frac{12m}{r}(e^{at} - 1) \\ S(t) = \frac{S_n e^{at} - \frac{12m}{r}(e^{at} - 1) \\ S(t) = \frac{S_n e^{at} - \frac{12m}{r}(e^{at} - 1) \\ S(t) = \frac{S_n e^{at} - \frac{12m}{r}(e^{at} - 1) \\ S(t) = \frac{S_n e^{at} - \frac{12m}{r}(e^{at} - 1) \\ S(t) = \frac{S_n e^{at} - \frac{12m}{r}(e^{at} - 1) \\ S(t) = \frac{S_n e^{at} - \frac{12m}{r}(e^{at} - 1) \\ S(t) = \frac{S_n e^{at} - \frac{12m}{r}(e^{at} - 1) \\ S(t) = \frac{S_n e^{at} - \frac{12m}{r}(e^{at} - 1) \\ S(t) = \frac{S_n e^{at} - \frac{12m}{r}(e^{at} - 1) \\ S(t) = \frac{S_n e^{at} - \frac{12m}{r}(e^{at} - 1) \\ S(t) = \frac{S_n e^{at} - \frac{12m}{r}(e^{at} - 1) \\ S(t) = \frac{S_n e^{at} - \frac{12m}{r}(e^{at} - 1) \\ S(t) = \frac{S_n e^{at} - \frac{12m}{r}(e^{at} - $ | $r\left[\frac{1}{\sec}\right] = growth \ rate, \ r > 0$ | | | |
| • Population model (mice-owl) $p(t)[mice] population \frac{dp}{dt} = rp - k r \left[\frac{1}{year}\right] = \operatorname{growth rate} \\ r \left[\frac{1}{year}\right] = \operatorname{growth rate} \\ \frac{mice}{geowth rate} \\ \frac{mice}{geo$ | $\frac{dp}{dt} = rp - k$ $r \left[\frac{d}{dx} \right] = rp - rp$ | | $\frac{dQ}{dr} = -\frac{r}{Q}Q + c_{in}r$ | | |
| $p(t)[nicc] population \qquad \frac{dp}{dt} = rp - k$ $r\left[\frac{l}{ year}\right] = \underset{reproduction rate}{growth rate}$ $r\left[\frac{l}{ year}\right] = \underset{reproduction rate}{growth rate}$ $k\left[\frac{micc}{ year}\right] = \underset{outliness rate}{of killing micc}$ $p(\theta) = p_{\theta} [micc] initial population$ Bank model $S(t) [S] = investment or debt$ $r\left[\frac{l}{ year}\right] = \underset{rate of return}{amual interest rate}$ $k\left[\frac{S}{ year}\right] = \underset{rate of return}{of deposits} k = 12 \cdot m \frac{dS}{dt} = rS + k S(t) = S_{\theta}e^{rt} + \frac{k}{r}(e^{rt} - 1) S(t) = S_{\theta}e^{rt} + \frac{12m}{r}(e^{rt} - 1)$ $w\left[\frac{S}{ year}\right] = \underset{rate of vithdrawls}{outinous annual} w = 12 \cdot m \frac{dS}{dt} = rS - w S(t) = S_{\theta}e^{rt} - \frac{w}{r}(e^{rt} - 1) S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $p\left[\frac{S}{ year}\right] = \underset{rate of poyments}{outinous annual} p = 12 \cdot m \frac{dS}{dt} = rS - p S(t) = S_{\theta}e^{rt} - \frac{p}{r}(e^{rt} - 1) S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $m\left[\frac{S}{ month }\right] = \underset{withdrawls}{monthy deposits} p = 12 \cdot m \frac{dS}{dt} = rS - p S(t) = S_{\theta}e^{rt} - \frac{p}{r}(e^{rt} - 1) S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $m\left[\frac{S}{ month }\right] = \underset{withdrawls}{monthy deposits} g = 12 \cdot m \frac{dS}{dt} = rS - p S(t) = S_{\theta}e^{rt} - \frac{p}{r}(e^{rt} - 1) S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $m\left[\frac{S}{ month }\right] = \underset{withdrawls}{monthy deposits} g = 12 \cdot m \frac{dS}{dt} = rS - p S(t) = S_{\theta}e^{rt} - \frac{p}{r}(e^{rt} - 1) S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $m\left[\frac{S}{ month }\right] = \underset{withdrawls}{monthy deposits} g = \frac{12 \cdot m}{r} \frac{dS}{dt} = rS - p S(t) = S_{\theta}e^{rt} - \frac{p}{r}(e^{rt} - 1) S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $m\left[\frac{S}{ month }\right] = \underset{withdrawls}{monthy deposits} g = \frac{12 \cdot m}{r} \frac{dS}{dt} = -k \cdot (u - T)$ | $p(t)[m(x) \text{ perplation} \qquad \qquad$ | | dt V = m | | |
| $p(t)[nicc] population \qquad \frac{dp}{dt} = rp - k$ $r\left[\frac{l}{ year}\right] = \underset{reproduction rate}{growth rate}$ $r\left[\frac{l}{ year}\right] = \underset{reproduction rate}{growth rate}$ $k\left[\frac{micc}{ year}\right] = \underset{outliness rate}{of killing micc}$ $p(\theta) = p_{\theta} [micc] initial population$ Bank model $S(t) [S] = investment or debt$ $r\left[\frac{l}{ year}\right] = \underset{rate of return}{amual interest rate}$ $k\left[\frac{S}{ year}\right] = \underset{rate of return}{of deposits} k = 12 \cdot m \frac{dS}{dt} = rS + k S(t) = S_{\theta}e^{rt} + \frac{k}{r}(e^{rt} - 1) S(t) = S_{\theta}e^{rt} + \frac{12m}{r}(e^{rt} - 1)$ $w\left[\frac{S}{ year}\right] = \underset{rate of vithdrawls}{outinous annual} w = 12 \cdot m \frac{dS}{dt} = rS - w S(t) = S_{\theta}e^{rt} - \frac{w}{r}(e^{rt} - 1) S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $p\left[\frac{S}{ year}\right] = \underset{rate of poyments}{outinous annual} p = 12 \cdot m \frac{dS}{dt} = rS - p S(t) = S_{\theta}e^{rt} - \frac{p}{r}(e^{rt} - 1) S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $m\left[\frac{S}{ month }\right] = \underset{withdrawls}{monthy deposits} p = 12 \cdot m \frac{dS}{dt} = rS - p S(t) = S_{\theta}e^{rt} - \frac{p}{r}(e^{rt} - 1) S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $m\left[\frac{S}{ month }\right] = \underset{withdrawls}{monthy deposits} g = 12 \cdot m \frac{dS}{dt} = rS - p S(t) = S_{\theta}e^{rt} - \frac{p}{r}(e^{rt} - 1) S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $m\left[\frac{S}{ month }\right] = \underset{withdrawls}{monthy deposits} g = 12 \cdot m \frac{dS}{dt} = rS - p S(t) = S_{\theta}e^{rt} - \frac{p}{r}(e^{rt} - 1) S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $m\left[\frac{S}{ month }\right] = \underset{withdrawls}{monthy deposits} g = \frac{12 \cdot m}{r} \frac{dS}{dt} = rS - p S(t) = S_{\theta}e^{rt} - \frac{p}{r}(e^{rt} - 1) S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $m\left[\frac{S}{ month }\right] = \underset{withdrawls}{monthy deposits} g = \frac{12 \cdot m}{r} \frac{dS}{dt} = -k \cdot (u - T)$ | $p(t)[m(x) \text{ perplation} \qquad \qquad$ | Population model (mice-owl) | | | |
| $r\left[\frac{1}{ yer }\right] = \frac{growthrate}{reproduction rate}$ $k\left[\frac{nice}{ yerr }\right] = \frac{growthrate}{reproduction rate}$ $k\left[\frac{mice}{ yerr }\right] = \frac{continuous rate}{ofkilling mice}$ $p(0) = p_{n} [mice] initial population$ Bank model $S(t) [S] = investment or debt$ $r\left[\frac{1}{ yerr }\right] = \frac{annual}{interest rate}$ $k\left[\frac{S}{ yerr }\right] = \frac{continuous annual}{rate of return}$ $k\left[\frac{S}{ yerr }\right] = \frac{continuous annual}{rate of deposits}$ $k = 12 \cdot m$ $\frac{dS}{dt} = rS + k$ $S(t) = S_{0}e^{rt} + \frac{k}{r}(e^{rt} - 1)$ $S(t) = S_{0}e^{rt} + \frac{12m}{r}(e^{rt} - 1)$ $w\left[\frac{S}{ yerr }\right] = \frac{continuous annual}{rate of payments}$ $p = 12 \cdot m$ $\frac{dS}{dt} = rS - p$ $S(t) = S_{0}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $p\left[\frac{S}{ yerr }\right] = \frac{continuous annual}{rate of payments}$ $p = 12 \cdot m$ $\frac{dS}{dt} = rS - p$ $S(t) = S_{0}e^{rt} - \frac{p}{r}(e^{rt} - 1)$ $S(t) = S_{0}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $m\left[\frac{S}{month}\right] = \frac{monthly deposits}{monthly deposits,}$ $S(t) = S_{0}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $m\left[\frac{S}{month}\right] = \frac{monthly deposits}{monthly deposits,}$ $S(t) = S_{0}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $m\left[\frac{S}{month}\right] = \frac{monthly deposits}{monthly deposits,}$ $S(t) = (S_{0} - \frac{12m}{r})e^{rt} + \frac{12m}{r}$ $S(0) = S_{n} [S] initial deposit or a loan$ | $r\left[\frac{1}{year}\right] = growth rate regreduction rate k\left[\frac{mixe}{year}\right] = growth rate continuous rate p(0) = p_{c}\left[mixe\right] initial population B Bank model S(t) [S] = investment or debt r \left[\frac{1}{year}\right] = annual interest rate rate of depasits k = 12 · m \frac{dS}{dt} = rS + k S(t) = S_{p}e^{rt} + \frac{k}{r}(e^{rt} - 1) S(t) = S_{p}e^{rt} + \frac{12m}{r}(e^{rt} - 1)w\left[\frac{s}{year}\right] = continuous annual w \left[\frac{s}{year}\right] = continuous annual w = 12 · m \frac{dS}{dt} = rS - w S(t) = S_{p}e^{rt} - \frac{w}{r}(e^{rt} - 1) S(t) = S_{p}e^{rt} - \frac{12m}{r}(e^{rt} - 1)w\left[\frac{s}{year}\right] = continuous annual w = 12 · m \frac{dS}{dt} = rS - w S(t) = S_{p}e^{rt} - \frac{w}{r}(e^{rt} - 1) S(t) = S_{p}e^{rt} - \frac{12m}{r}(e^{rt} - 1)p\left[\frac{s}{year}\right] = continuous annual p = 12 · m \frac{dS}{dt} = rS - p S(t) = S_{p}e^{rt} - \frac{p}{r}(e^{rt} - 1) S(t) = S_{p}e^{rt} - \frac{12m}{r}(e^{rt} - 1)m\left[\frac{s}{month}\right] = monthy deposits. S(t) = S_{p}e^{rt} - \frac{12m}{r}(e^{rt} - 1) S(t) = S_{p}e^{rt} + \frac{12m}{r}S(t) = (S_{0} - \frac{12m}{r})e^{rt} + \frac{12m}{r}S(t) = (S_{0} - \frac{12m}{r})e^{rt}$ | | $\frac{dp}{dt} = rp - k$ | | |
| $p(\theta) = p_{e} [mice] \text{ initial population}$ $= \text{ Bank model}$ $S(t) [S] = investment or debt$ $r \left[\frac{1}{year}\right] = annual interest rate rate of return$ $k \left[\frac{S}{year}\right] = continuous annual rate of deposits k = 12 \cdot m$ $\frac{dS}{dt} = rS + k$ $S(t) = S_{\theta}e^{rt} + \frac{k}{r}(e^{rt} - 1)$ $S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $w \left[\frac{S}{year}\right] = continuous annual of the second deposits w = 12 \cdot m$ $\frac{dS}{dt} = rS - w$ $S(t) = S_{\theta}e^{rt} - \frac{W}{r}(e^{rt} - 1)$ $S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $p \left[\frac{S}{year}\right] = continuous annual of the second deposits p = 12 \cdot m$ $\frac{dS}{dt} = rS - p$ $S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $m \left[\frac{S}{year}\right] = continuous annual of the second deposits p = 12 \cdot m$ $\frac{dS}{dt} = rS - p$ $S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $m \left[\frac{S}{year}\right] = monthly deposits,$ $S(t) = \left(S_{\theta} - \frac{12m}{r}\right)e^{rt} + \frac{12m}{r}$ $S(t) = \left(S_{\theta} - \frac{12m}{r}\right)e^{rt}$ | $p(\theta) = p_{k} [mice] initial population$ $= \text{Bank model}$ $S(t) [S] = investment or debt$ $r \left[\frac{1}{year} \right] = annual interest rate$ $k \left[\frac{S}{year} \right] = continuous annual interest rate is rate of return $ $k \left[\frac{S}{year} \right] = continuous annual interest rate is rate of depositis $ $k = t2 \cdot m$ $\frac{dS}{dt} = rS + k$ $S(t) = S_{\theta}e^{rt} + \frac{k}{r}(e^{rt} - 1)$ $S(t) = S_{\theta}e^{rt} + \frac{12m}{r}(e^{rt} - 1)$ $w \left[\frac{S}{year} \right] = continuous annual is w = t2 \cdot m$ $\frac{dS}{dt} = rS - w$ $S(t) = S_{\theta}e^{rt} - \frac{m}{r}(e^{rt} - 1)$ $S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $p \left[\frac{S}{year} \right] = continuous annual is w = t2 \cdot m$ $\frac{dS}{dt} = rS - p$ $S(t) = S_{\theta}e^{rt} - \frac{m}{r}(e^{rt} - 1)$ $S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $m \left[\frac{S}{monh} \right] = monthly deposits,$ $S(t) = \frac{S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)}{s}$ $S(t) = \frac{S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - \frac{12m}{r}(e^{rt} - 1)}{s}$ $S(t) = \frac{S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - \frac{12m}{r}(e^{rt} - \frac{12m}{r}(e^{rt} - \frac{12m}{r}(e^{rt} - \frac{12m}{r}(e^{rt} - \frac{12m}{r$ | $r\left[\frac{1}{year}\right] = \frac{growth\ rate}{reproduction\ rate}$ | dt - | | |
| a Bank model S(t) [S] = investment or debt $r\left[\frac{1}{y e a r}\right] = \stackrel{annual interest rate}{rate of return}$ $k\left[\frac{S}{y e a r}\right] = \stackrel{continuous annual}{rate of deposits} k = 12 \cdot m \frac{dS}{dt} = rS + k S(t) = S_0 e^{rt} + \frac{k}{r} (e^{rt} - 1) S(t) = S_0 e^{rt} + \frac{12m}{r} (e^{rt} - 1)$ $w\left[\frac{S}{y e a r}\right] = \stackrel{continuous annual}{rate of deposits} w = 12 \cdot m \frac{dS}{dt} = rS - w S(t) = S_0 e^{rt} - \frac{12m}{r} (e^{rt} - 1)$ $p\left[\frac{S}{y e a r}\right] = \stackrel{continuous annual}{rate of payments} p = 12 \cdot m \frac{dS}{dt} = rS - p S(t) = S_0 e^{rt} - \frac{p}{r} (e^{rt} - 1) S(t) = S_0 e^{rt} - \frac{12m}{r} (e^{rt} - 1)$ $m\left[\frac{S}{month}\right] = \stackrel{monthly deposits,}{withdrawls or payments} S(t) = S_0 e^{rt} - \frac{12m}{r} (e^{rt} - 1) S(t) = S_0 e^{rt} + \frac{12m}{r}$ $S(0) = S_0 [S] initial deposit or a loan$ | ■ Bank model $S(t) [S] = investment or debt$ $r \left[\frac{1}{ yaar }\right] = annual interest rate rate of return k \left[\frac{S}{ yaar }\right] = continuous annual rate of deposits k = 12 · m \frac{dS}{dt} = rS + k S(t) = S_0 e^{rt} + \frac{k}{r} (e^{rt} - 1) S(t) = S_0 e^{rt} - \frac{12m}{r} (e^{rt} - 1) w \left[\frac{S}{ yaar }\right] = continuous annual w = 12 · m \frac{dS}{dt} = rS - w S(t) = S_0 e^{rt} - \frac{w}{r} (e^{rt} - 1) S(t) = S_0 e^{rt} - \frac{12m}{r} (e^{rt} - 1) p \left[\frac{S}{ yaar }\right] = continuous annual p = 12 · m \frac{dS}{dt} = rS - p S(t) = S_0 e^{rt} - \frac{p}{r} (e^{rt} - 1) S(t) = S_0 e^{rt} - \frac{12m}{r} (e^{rt} - 1) m \left[\frac{S}{monh}\right] = monthly deposits, S(t) = S_0 e^{rt} - \frac{p}{r} (e^{rt} - 1) S(t) = S_0 e^{rt} - \frac{12m}{r} (e^{rt} - 1) S(t) = S_0 e^{rt} - \frac{12m}{r} (e^{rt} - 1) S(t) = S_0 e^{rt} - \frac{12m}{r} (e^{rt} - 1) m \left[\frac{S}{monh}\right] = monthly deposits, S(t) = S_0 e^{rt} - \frac{12m}{r} (e^{rt} - 1) S(t) = $ | $k\left[\frac{mice}{year}\right] = \frac{continuous\ rate}{of\ killing\ mice}$ | | | |
| $S(t) [S] = investment or debt$ $r \left[\frac{l}{year} \right] = annual interest rate$ $r \left[\frac{l}{year} \right] = annual interest rate$ $k \left[\frac{S}{year} \right] = continuous annual$ $k = 12 \cdot m$ $\frac{dS}{dt} = rS + k$ $S(t) = S_0 e^{rt} + \frac{k}{r} (e^{rt} - 1)$ $S(t) = S_0 e^{rt} + \frac{12m}{r} (e^{rt} - 1)$ $w \left[\frac{S}{year} \right] = continuous annual$ $w = 12 \cdot m$ $\frac{dS}{dt} = rS - w$ $S(t) = S_0 e^{rt} - \frac{m}{r} (e^{rt} - 1)$ $S(t) = S_0 e^{rt} - \frac{12m}{r} (e^{rt} - 1)$ $p \left[\frac{S}{year} \right] = continuous annual$ $rate of withdrawls$ $p = 12 \cdot m$ $\frac{dS}{dt} = rS - p$ $S(t) = S_0 e^{rt} - \frac{p}{r} (e^{rt} - 1)$ $S(t) = S_0 e^{rt} - \frac{12m}{r} (e^{rt} - 1)$ $m \left[\frac{S}{month} \right] = monthly deposits,$ $S(t) = S_0 [S] initial deposit or a loan$ $S(t) = S_0 [S] initial deposit or a loan$ | $S(t) [S] = investment or debt$ $r \left[\frac{l}{yaar}\right] = annual interest rate$ $r \left[\frac{l}{yaar}\right] = continuous annual$ $k = 12 \cdot m$ $\frac{dS}{dt} = rS + k$ $S(t) = S_{\theta}e^{rt} + \frac{k}{r}(e^{rt} - 1)$ $S(t) = S_{\theta}e^{rt} + \frac{12m}{r}(e^{rt} - 1)$ $w \left[\frac{s}{year}\right] = continuous annual$ $w = 12 \cdot m$ $\frac{dS}{dt} = rS - w$ $S(t) = S_{\theta}e^{rt} - \frac{w}{r}(e^{rt} - 1)$ $S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $p \left[\frac{s}{year}\right] = continuous annual$ $p = 12 \cdot m$ $\frac{dS}{dt} = rS - p$ $S(t) = S_{\theta}e^{rt} - \frac{p}{r}(e^{rt} - 1)$ $S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $m \left[\frac{s}{month}\right] = continuous annual$ $p = 12 \cdot m$ $\frac{dS}{dt} = rS - p$ $S(t) = S_{\theta}e^{rt} - \frac{p}{r}(e^{rt} - 1)$ $S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $S(t) = S_{\theta}e^{rt} -$ | $p(\theta) = p_{\theta}$ [mice] initial population | | | |
| $r\left[\frac{1}{year}\right] = annual interest raterate of return k \left[\frac{s}{year}\right] = continuous annualrate of deposits k = 12 \cdot m \frac{dS}{dt} = rS + k S(t) = S_0e^{rt} + \frac{k}{r}(e^{rt} - 1) S(t) = S_0e^{rt} + \frac{12m}{r}(e^{rt} - 1)w\left[\frac{s}{year}\right] = continuous annualrate of withdrawls w = 12 \cdot m \frac{dS}{dt} = rS - w S(t) = S_0e^{rt} - \frac{w}{r}(e^{rt} - 1) S(t) = S_0e^{rt} - \frac{12m}{r}(e^{rt} - 1)p\left[\frac{s}{year}\right] = continuous annualrate of payments p = 12 \cdot m \frac{dS}{dt} = rS - p S(t) = S_0e^{rt} - \frac{p}{r}(e^{rt} - 1) S(t) = S_0e^{rt} - \frac{12m}{r}(e^{rt} - 1)m\left[\frac{s}{month}\right] = monthly deposits,S(t) = S_0 [s] initial deposit or a loanI Newton's Lawu(t)[^{r}F] temperature \frac{du}{dt} = -k \cdot (u - T)$ | $r\left[\frac{l}{ year}\right] = annual interest raterate of return k\left[\frac{s}{ year}\right] = continuous annual k = 12 \cdot m \frac{dS}{dt} = rS + k$ $S(t) = S_{\theta}e^{rt} + \frac{k}{r}(e^{rt} - 1)$ $S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $w\left[\frac{s}{ year}\right] = continuous annual rate of withdrawls w = 12 \cdot m$ $\frac{dS}{dt} = rS - w$ $S(t) = S_{\theta}e^{rt} - \frac{w}{r}(e^{rt} - 1)$ $S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $p\left[\frac{s}{ year}\right] = continuous annual rate of payments rate of payments p = 12 \cdot m$ $\frac{dS}{dt} = rS - p$ $S(t) = S_{\theta}e^{rt} - \frac{p}{r}(e^{rt} - 1)$ $S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $m\left[\frac{s}{month}\right] = monthly deposits,$ $S(t) = S_{\theta}[s] initial deposit or a loan$ $S(t) = S_{\theta}[s] initial deposit or a loan$ $s(t) = -k \cdot (u - T)$ $k\left[\frac{t}{ sc }\right] = \frac{t}{r}$ $\tau [sc] = \frac{p_{k}L}{h} > 0 \text{ time constant}$ $\frac{du}{r} = -k \cdot (u - T)$ $k\left[\frac{t}{s}\right] velocity$ $m \oint F_{t} = -yv$ $v(t) \left[\frac{f}{s}\right] velocity$ $m \oint F_{t} = mg$ $\frac{dv}{dt} = -\frac{y}{r}v + g$ | Bank model | | | |
| $r\left[\frac{1}{year}\right] = annual interest raterate of return k \left[\frac{s}{year}\right] = continuous annualrate of deposits k = 12 \cdot m \frac{dS}{dt} = rS + k S(t) = S_0e^{rt} + \frac{k}{r}(e^{rt} - 1) S(t) = S_0e^{rt} + \frac{12m}{r}(e^{rt} - 1)w\left[\frac{s}{year}\right] = continuous annualrate of withdrawls w = 12 \cdot m \frac{dS}{dt} = rS - w S(t) = S_0e^{rt} - \frac{w}{r}(e^{rt} - 1) S(t) = S_0e^{rt} - \frac{12m}{r}(e^{rt} - 1)p\left[\frac{s}{year}\right] = continuous annualrate of payments p = 12 \cdot m \frac{dS}{dt} = rS - p S(t) = S_0e^{rt} - \frac{p}{r}(e^{rt} - 1) S(t) = S_0e^{rt} - \frac{12m}{r}(e^{rt} - 1)m\left[\frac{s}{month}\right] = monthly deposits,S(t) = S_0 [s] initial deposit or a loanI Newton's Lawu(t)[^{r}F] temperature \frac{du}{dt} = -k \cdot (u - T)$ | $r\left[\frac{l}{ year}\right] = annual interest rate rate of return k \left[\frac{s}{ year}\right] = continuous annual rate of deposits k = 12 \cdot m dS = rS + k S(t) = S_{\theta}e^{rt} + \frac{k}{r}(e^{rt} - 1) S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1)$ $w\left[\frac{s}{ year}\right] = continuous annual rate of withdrawls w = 12 \cdot m dS = rS - w S(t) = S_{\theta}e^{rt} - \frac{m}{r}(e^{rt} - 1)$ S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1) $p\left[\frac{s}{ year}\right] = continuous annual rate of payments p = 12 \cdot m dS = rS - p S(t) = S_{\theta}e^{rt} - \frac{p}{r}(e^{rt} - 1)$ S(t) = S_{\theta}e^{rt} - \frac{12m}{r}(e^{rt} - 1) $m\left[\frac{s}{month}\right] = monthly deposits, S(t) = S_{\theta}[s] initial deposit or a loan S(t) = S_{\theta}[s] initial deposit or a loan Newton's Law u(t)[^{T}F] temperature k \left[\frac{t}{lsc}\right] = \frac{t}{r}, r [sc] = \frac{p_{c}L^{r}}{h_{A}} > 0 time constant m \oint_{F_{x}} F_{y} = ny \frac{dv}{dt} = -k \cdot (u - T)$ | S(t) [\$] = investment or debt | | | |
| $k \begin{bmatrix} \frac{s}{year} \end{bmatrix} = \stackrel{\text{continuous annual}}{rate of deposits} \qquad k = 12 \cdot m \qquad \frac{dS}{dt} = rS + k \qquad S(t) = S_0 e^{rt} + \frac{k}{r} (e^{rt} - 1) \qquad S(t) = S_0 e^{rt} + \frac{12m}{r} (e^{rt} - 1) \qquad S(t) = S_0 e^{rt} - \frac{12m}{r} (e^{rt} - 1) \qquad S(t) = S_0 e^{$ | $k \begin{bmatrix} \frac{s}{year} \end{bmatrix} = \begin{array}{c} continuous annual \\ rate of deposits \\ k = 12 \cdot m \\ \hline dt = rS + k \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}{r}) \left(e^{rt} - 1\right) \\ \hline star = of (upper + \frac{k}$ | | | | |
| $\begin{bmatrix} y \text{ end } y \text{ for a legende} & u \text{ for legende} & u \text{ for legende} & u \text{ for a legende} & u for a $ | $w\left[\frac{s}{year}\right] = to to g a point \qquad \text{if } \qquad t \qquad$ | $r\left[\frac{1}{year}\right] = rate of return$ | | | |
| $p\left[\frac{s}{year}\right] = \frac{continuous\ annual}{rate\ of\ payments} \qquad p = 12 \cdot m \qquad \frac{dS}{dt} = rS - p \qquad S(t) = S_0 e^{rt} - \frac{p}{r} \left(e^{rt} - 1\right) \qquad S(t) = S_0 e^{rt} - \frac{12m}{r} \left(e^{rt} - 1\right)$ $m\left[\frac{s}{month}\right] = \frac{monthly\ deposits,}{withdrawls\ or\ payments} \qquad S(t) = \left(S_0 - \frac{12m}{r}\right) e^{rt} + \frac{12m}{r}$ $S(0) = S_0 [s]\ initial\ deposit\ or\ a\ loan$ $\bullet Newton's\ Law$ $u(t)[^oF]\ temperature \qquad \frac{du}{dt} = -k \cdot (u - T)$ | $p \left[\frac{s}{year}\right] = \begin{array}{c} continuous annual \\ rate of payments \end{array} \qquad p = 12 \cdot m \qquad \frac{dS}{dt} = rS - p \qquad S(t) = S_0 e^{rt} - \frac{p}{r} (e^{rt} - 1) \qquad S(t) = S_0 e^{rt} - \frac{12m}{r} (e^{rt} - 1) \\ m \left[\frac{s}{month}\right] = \begin{array}{c} monthly deposits, \\ s(t) = \left(S_0 - \frac{12m}{r}\right) e^{rt} + \frac{12m}{r} \\ S(0) = S_0 \left[S\right] initial deposit or a loan \end{array}$ • Newton's Law $u(t) \left[{}^{*}F_{}\right] temperature \qquad \frac{du}{dt} = -k \cdot (u - T) \\ k \left[\frac{1}{sec}\right] = \frac{1}{r} \\ r \left[\sec\right] = \frac{pc_s V}{hA} > 0 \ time constant \end{aligned}$ • Falling bal $v(t) \left[\frac{f}{s}\right] velocity \qquad m \oint \begin{array}{c} F_f = -\gamma v \\ F_s = mg \qquad \frac{dv}{dt} = -\frac{\gamma}{m} v + g \end{array}$ | $k\left[\frac{\$}{year}\right] = \frac{continuous\ annual}{rate\ of\ deposits} \qquad k = 12 \cdot m$ | $\frac{dS}{dt} = rS + k$ | $S(t) = S_0 e^{rt} + \frac{k}{r} \left(e^{rt} - 1 \right)$ | $S(t) = S_0 e^{rt} + \frac{12m}{r} \left(e^{rt} - I \right)$ |
| $m \left[\frac{\$}{month}\right] = \frac{monthly deposits,}{with drawls or payments}$ $S(t) = \left(S_0 - \frac{12m}{r}\right)e^{rt} + \frac{12m}{r}$ $S(0) = S_0 \left[\$\right] \text{ initial deposit or a loan}$ $\mathbf{Newton's Law}$ $u(t) \left[{}^{\circ}F\right] \text{ temperature}$ $\frac{du}{dt} = -k \cdot (u - T)$ | $m \begin{bmatrix} s \\ month \end{bmatrix} = monthly deposits,$ $s(t) = \left(S_0 - \frac{12m}{r}\right)e^{rt} + \frac{12m}{r}$ $u(t)[^{\circ}F] \text{ imperature}$ $u(t)[^{\circ}F] \text{ imperature}$ $k \begin{bmatrix} \frac{1}{\sec} \end{bmatrix} = \frac{1}{r}$ $r [\sec] = \frac{\rho c_F V}{hA} > 0 \text{ time constant}$ $F_f = -\gamma v$ $v(t) \begin{bmatrix} \frac{n}{s} \end{bmatrix} \text{ velocity}$ $m \oint F_s = mg$ $\frac{dv}{dt} = -\frac{\gamma}{m}v + g$ | $w\left[\frac{\$}{year}\right] = \frac{continuous\ annual}{rate\ of\ withdrawls} \qquad w = 12 \cdot m$ | $\frac{dS}{dt} = rS - w$ | $S(t) = S_0 e^{rt} - \frac{w}{r} \left(e^{rt} - l \right)$ | $S(t) = S_0 e^{rt} - \frac{12m}{r} \left(e^{rt} - l \right)$ |
| $S(t) = \left(S_0 - \frac{1}{r}\right)e^{t} + \frac{1}{r}$ $S(t) = \left(S_0 - \frac{1}{r}\right)e^{t} + \frac{1}{r}$ Newton's Law $u(t)[{}^{\circ}F] \text{ temperature} \qquad \frac{du}{dt} = -k \cdot (u - T)$ | $S(t) = \begin{bmatrix} S_0 - \frac{1}{r} \end{bmatrix} e^{r} + \frac{1}{r}$ $S(t) = \begin{bmatrix} S_0 - \frac{1}{r} \end{bmatrix} e^{r} + \frac{1}{r}$ $S(t) = \begin{bmatrix} S_0 - \frac{1}{r} \end{bmatrix} e^{r} + \frac{1}{r}$ $u(t) \begin{bmatrix} r \end{bmatrix} temperature$ $\frac{du}{dt} = -k \cdot (u - T)$ $k \begin{bmatrix} \frac{1}{sc} \end{bmatrix} = \frac{1}{r}$ $\tau \begin{bmatrix} sc \end{bmatrix} = \frac{\rho c_p V}{hA} > 0 time \ constant$ $F_{alling \ ball}$ $v(t) \begin{bmatrix} \frac{f}{s} \end{bmatrix} velocity$ $m \oint F_{s} = mg$ $\frac{dv}{dt} = -\frac{\gamma}{m}v + g$ | $p\left[\frac{\$}{year}\right] = \frac{continuous\ annual}{rate\ of\ payments} \qquad p = 12 \cdot m$ | $\frac{dS}{dt} = rS - p$ | $S(t) = S_0 e^{rt} - \frac{p}{r} \left(e^{rt} - l \right)$ | $S(t) = S_0 e^{rt} - \frac{12m}{r} \left(e^{rt} - 1 \right)$ |
| $S(0) = S_0 [S] \text{ initial deposit or a loan}$ $\blacksquare \text{ Newton's Law}$ $u(t)[^{\circ}F] \text{ temperature} \qquad \qquad \frac{du}{dt} = -k \cdot (u - T)$ | $S(0) = S_0 [\$] \text{ initial deposit or a loan}$ $\bullet \text{ Newton's Law}$ $u(t) [\degree F] \text{ temperature} \qquad \qquad \frac{du}{dt} = -k \cdot (u - T)$ $k \left[\frac{1}{\sec}\right] = \frac{1}{\tau}$ $\tau [\sec] = \frac{\rho c_p V}{hA} > 0 \text{ time constant}$ $\bullet \text{ Falling ball}$ $v(t) \left[\frac{f}{s}\right] \text{ velocity} \qquad \qquad m \oint \begin{array}{c} F_f = -\gamma v \\ F_g = mg \end{array} \qquad \qquad \frac{dv}{dt} = -\frac{\gamma}{m} v + g$ | $m\left[\frac{\$}{month}\right] = \frac{monthly\ deposits,}{withdrawls\ or\ payments}$ | | | $S(t) = \left(S_0 - \frac{12m}{r}\right)e^{rt} + \frac{12m}{r}$ |
| $u(t)[{}^{\circ}F] \ temperature \qquad \qquad \frac{du}{dt} = -k \cdot (u - T)$ | $u(t)[{}^{\circ}F] \text{ temperature} \qquad \qquad \frac{du}{dt} = -k \cdot (u - T)$ $k \left[\frac{l}{\sec}\right] = \frac{l}{\tau}$ $\tau [\sec] = \frac{\rho c_p V}{hA} > 0 \text{ time constant}$ $\mathbf{I} \mathbf{Falling ball}$ $v(t) \left[\frac{ft}{s}\right] \text{ velocity} \qquad \qquad m \oint \begin{array}{c} F_f = -\gamma v \\ F_g = mg \end{array} \qquad \qquad \frac{dv}{dt} = -\frac{\gamma}{m} v + g$ | $S(0) = S_0$ [\$] initial deposit or a loan | | | |
| $u(t)[{}^{\circ}F] \ temperature \qquad \qquad \frac{du}{dt} = -k \cdot (u - T)$ | $u(t)[{}^{\circ}F] \text{ temperature} \qquad \qquad \frac{du}{dt} = -k \cdot (u - T)$ $k \left[\frac{l}{\sec}\right] = \frac{l}{\tau}$ $\tau [\sec] = \frac{\rho c_p V}{hA} > 0 \text{ time constant}$ $\mathbf{I} \mathbf{Falling ball}$ $v(t) \left[\frac{ft}{s}\right] \text{ velocity} \qquad \qquad m \oint \begin{array}{c} F_f = -\gamma v \\ F_g = mg \end{array} \qquad \qquad \frac{dv}{dt} = -\frac{\gamma}{m} v + g$ | ■ Newton's Law | | | |
| $k\left[\frac{1}{2}\right] = \frac{1}{2}$ | $\tau \left[\sec \right] = \frac{\rho c_p V}{hA} > 0 time \ constant$ $\bullet Falling \ ball$ $v(t) \left[\frac{fi}{s} \right] \ velocity$ $m \left[\begin{array}{c} f_f \\ F_g = mg \end{array} \right] \frac{dv}{dt} = -\frac{\gamma}{m}v + g$ | | $\frac{du}{dt} = -k \cdot (u - T)$ | | |
| Sec 7 | $\tau \left[\sec \right] = \frac{\rho c_p V}{hA} > 0 time \ constant$ $\bullet Falling \ ball$ $v(t) \left[\frac{fi}{s} \right] \ velocity$ $m \left[\begin{array}{c} f_f \\ F_g = mg \end{array} \right] \frac{dv}{dt} = -\frac{\gamma}{m}v + g$ | $k \left[\frac{I}{\sec} \right] = \frac{I}{\tau}$ | | | |
| | $v(t) \left[\frac{fi}{s}\right] velocity \qquad \qquad$ | | | | |
| Falling ball $F_f = -\gamma v$ | $I_g - mg$ | Falling ball $F_f = -\gamma v$ | | | |
| $\frac{dv}{dv} = -\frac{\gamma}{v} + g$ | $I_g - mg$ | $m \phi$ | $\frac{dv}{dt} = -\frac{\gamma}{v}v + \varphi$ | | |
| $v(t)\left[\frac{1}{s}\right] \text{ velocity} \qquad \qquad dt \qquad m$ | $v(0) = v_0 \left[\frac{ft}{s}\right]$ initial velocity | $v(t) \lfloor \frac{s}{s} \rfloor \text{ velocity}$ $F_g = mg$ | dt m | | |
| $v(\theta) = v_{\theta} \left[\frac{ft}{s}\right]$ initial velocity | | $v(0) = v_0 \left[\frac{ft}{s}\right]$ initial velocity | | | |

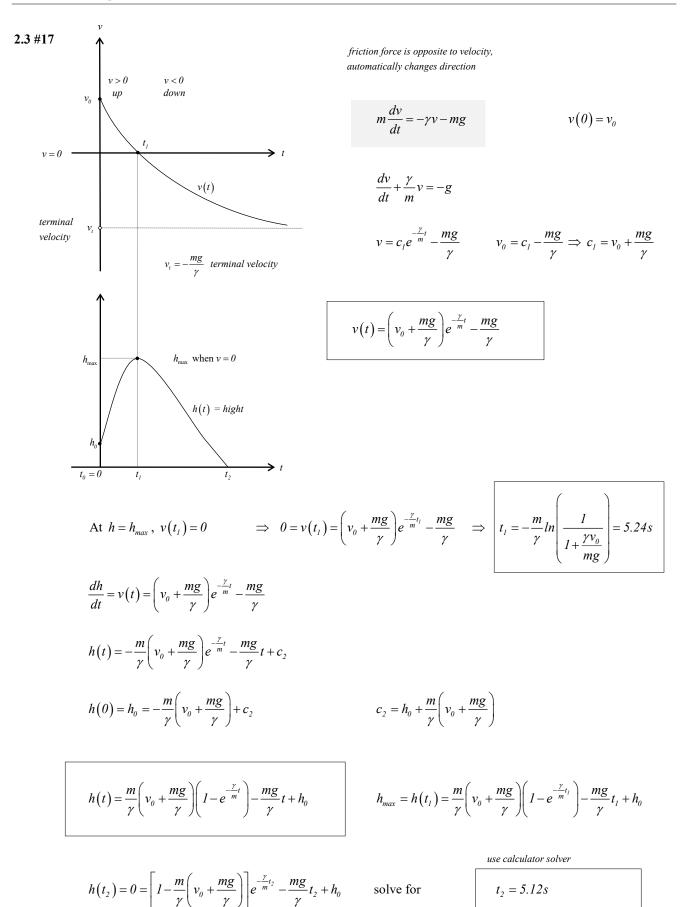


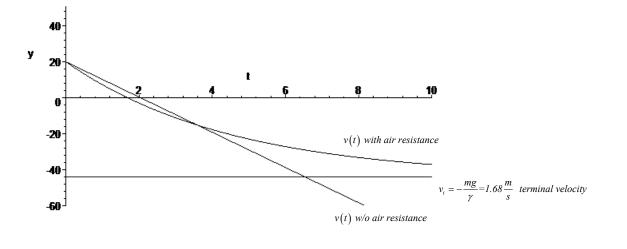
Integrals Expected Known

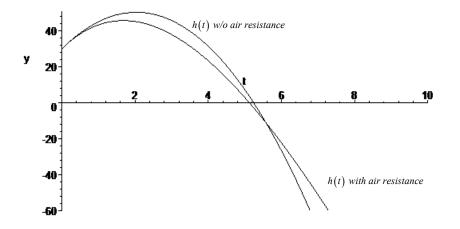


| 1. $\int \sec x dx$ | $= \ln \left \sec x + \tan x \right + c$ |
|---|---|
| 2. $\int \csc x dx$ | $= \ln \left \csc x - \cot x \right + c$ |
| 3. $\int \tan x dx$ | $= -\ln \left \cos x \right + c$ |
| 4. $\int \cot x dx$ | $= \ln \sin x + c$ |
| 5. $\int \ln x dx$ | $= x \ln x - x + c$ |
| $6. \int \frac{1}{a^2 + x^2} dx$ | $= \frac{1}{a} \arctan \frac{x}{a} + c$ |
| $7. \int \frac{1}{\sqrt{a^2 - x^2}} dx$ | $= \arcsin \frac{x}{a} + c$ |
| 8. $\int e^{ax} \sin bx dx$ | $= \frac{e^{ax} \left(a \sin bx - b \cos bx \right)}{a^2 + b^2} + c$ |
| 9. $\int e^{ax} \cos bx dx$ | $= \frac{e^{ax} \left(a \sin bx + b \cos bx \right)}{a^2 + b^2} + c$ |
| 10. $\int \sin^2 mx dx$ | $= \frac{mx - \sin mx \cos mx}{2m} + c$ |
| 11. $\int \cos^2 mx dx$ | $= \frac{mx + \sin mx \cos mx}{2m} + c$ |

2.3 Falling ball For convenience, assume that the **positive** direction is **UPWARD** (in contrast to textbook, see p.2) v > 0v < 0down up direction of friction force is opposite $F_f = -\gamma v$ to direction of the velocity v = 0т $v(t) = -gt + v_0$ gravitational force is $F_g = -mg$ directed downward (negative sign) $F = m \frac{dv}{dt}$ $F = F_g + F_f$ resulting force Newton's Law h_{\max} when v = 0 $m\frac{dv}{dt} = -mg - \gamma v$ $h_{\rm ma}$ governing equation for velocity v(t)integrate from 0 to t \Rightarrow h(t) = hight $\frac{dh}{dt} = v(t)$ $h(t) = h_0 + \int_0^t v(t) dt$ $\rightarrow t$ $t_0 = 0$ t_{I} 50 40 30 conservation of energy elevation 20 $mg(h_{max} - h_0) = \frac{mv_0^2}{2} \implies (h_{max} - h_0) = \frac{v_0^2}{2g}$ 10 , 3 $m\frac{dv}{dt} = \gamma v - mg$ $v(\theta) = v_0, \quad \gamma = \theta$ 2.3 #16 no friction force $\frac{dv}{dt} = -g$ $v_0 = -g \not t + c_1 \implies c_1 = v_0$ $v = -gt + c_1$ $v(t) = -gt + v_0$ At $h = h_{max}$, $v(t_1) = 0$ $\Rightarrow 0 = -gt_1 + v_0 \Rightarrow t_1 = \frac{v_0}{g}$ $\frac{dh}{dt} = v(t) = -gt + v_0 \qquad \Rightarrow \quad h(t) = -\frac{gt^2}{2} + v_0 t + c_2$ $h(0) = h_0 = -\frac{gt^2}{2} + y_0 t + c_2 \implies c_2 = h_0$ $h(t) = -\frac{gt^2}{2} + v_0 t + h_0 \qquad \qquad h(t_2) = 0 = -\frac{gt_1^2}{2} + v_0 t_1 + h_0 \qquad \Rightarrow \qquad t_2 = \frac{v_0}{g} + \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h_0}{g}}$







| Math-303 | Chapters | s 1-2 | 1 st Order ODE | | | September 20, 2019 2 |
|----------|----------|----------------|-------------------------------------|-------------------|------------------|----------------------------|
| 2.3 #18 | Part I | $(v > \theta)$ | $m\frac{dv}{dt} = -\gamma v^2 - mg$ | $0 \le t \le t_1$ | $v(0) = v_0 > 0$ | friction force is downward |
| | | | $h(t) = h_0 + \int_0^{t_1} v(t) dt$ | | | |
| | | | | | | |

Part II
$$(v < 0)$$
 $m \frac{dv}{dt} = +\gamma v^2 - mg$ $t \ge t_1$ $v(t_1) = 0$ friction force is upward
 $h(t) = h_{\max} + \int_{t_1}^{t_2} v(t) dt$